Radar and Electronic Warfare Principles for the Non-Specialist, 4th Edition

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Exercise Solutions

This document contains detailed solution sheets for the exercises at the end of the chapters. These exercises are designed to reinforce the concepts presented in the book and illustrate their application. The solutions are in Mathcad 14.0 with all applicable units. Each solution sheet contains the exercise, a step-by-step description of how to solve the exercise with references to the book, a Mathcad solution (Mathcad performing the calculations), and an "explicit" solution (the numbers in their correct places in the equations). The explicit solutions are in yellow highlighted regions with a border. Examples of the Mathcad and explicit solutions are shown below for the equation for a straight line.

\[
m \equiv 2 \quad x \equiv 3 \quad b \equiv 4
\]

Mathcad Solution

\[
y := m \cdot x + b = 10
\]

 Explicit Solution

\[
= 2 \cdot 3 + 4 = 10
\]

Most of the results for the Mathcad and explicit solutions are numerically identical. However, some of the explicit solutions have different precisions (number of significant digits) and/or numerical results that are slightly rounded up or down. Also, when a term in an equation has a value of one (1), sometimes it is not included in the explicit solution. The numerical result and units are correct, only the value of one (1) is missing.

Good learning and calculating - Paul J. Hannen
Exercise 2-1  What is the range associated with a time delay $\Delta t = 0.67$ msec? What is the time delay associated with the range to the moon (earth-to-moon range $R = 3.84 \times 10^8$ meters)?

We start by defining the values for all the terms and necessary constants.

$$\Delta t \equiv 0.67 \times 10^{-3} \text{ sec} \quad R_{\text{EM}} \equiv 3.84 \times 10^8 \text{ m} \quad c \equiv 3 \times 10^8 \frac{\text{m}}{\text{sec}}$$

The range associated with a time delay is given by Equation 2-1.

$$R := \frac{c \cdot \Delta t}{2} = 1.005 \times 10^5 \text{ m}$$

$$R = 100.5 \text{ km}$$

This equation can also be solved for the time delay associated with range.

$$\Delta t = \frac{2 \cdot R}{c}$$

$$\Delta t := \frac{2 \cdot R_{\text{EM}}}{c} = 2.56 \text{ s}$$
**Exercise 2-2** If a radar with transmit peak power $P_R = 1 \text{ MW}$ and antenna gain in the direction of the target $G_{RT} = 40 \text{ dBi}$ irradiates a target with a radar cross section $\sigma = 1 \text{ m}^2$ target at a radar-to-target range $R_{RT} = 500 \text{ km}$. If the radar transmit loss $L_{Rt} = 0.5 \text{ dB}$, what power density arrives back at the radar antenna?

We start by defining the values for all the terms.

$P_R \equiv 1 \times 10^6 \cdot \text{W} \quad G_{RT \_dB} \equiv 40 \quad \sigma \equiv 1 \cdot \text{m}^2 \quad R_{RT} \equiv 500 \times 10^3 \cdot \text{m} \quad L_{Rt \_dB} \equiv 0.5$

The power density arriving back at the radar antenna is given by Equation 2-6. But first since we are using an algebraic equation we will convert the antenna gain and radar transmit loss from decibels to absolute.

$$G_{RT} := 10^{\left(\frac{G_{RT \_dB}}{10}\right)} = 1 \times 10^4$$

$$L_{Rt} := 10^{\left(\frac{L_{Rt \_dB}}{10}\right)} = 1.1220$$

$$\frac{P_R \cdot G_{RT} \cdot \sigma}{(4\pi)^2 \cdot R_{RT}^4 \cdot L_{Rt}} = 9.0303 \times 10^{-16} \cdot \frac{\text{W}}{\text{m}^2}$$
Exercise 2-3  What is the receiver thermal noise, $N$ (Watts, dBW, and dBm), for the following receiver characteristics: noise figure $F_R = 6$ dB and receiver bandwidth $B_R = 500$ kHz?  What is the equivalent noise temperature, $T_e$ (Kelvin), and noise power added by an actual receiver over what is generated by an ideal receiver, $\Delta N$ (Watts, dBW, and dBm)?

We start by defining the values for all the terms and necessary constants.

\[
F_{R_{\text{dB}}} = 6 \quad B_R = 500 \times 10^3 \text{Hz} \quad T_0 = 290 \text{K} \quad k = 1.38 \times 10^{-23} \text{W} \cdot \text{sec} \cdot \text{K}^{-1}
\]

First convert the noise figure from decibels to absolute units.

\[
F_R := 10 \left( \frac{F_{R_{\text{dB}}}}{10} \right) = 3.9811
\]

The receiver thermal noise, $N$ (Watts), is given in Equation 2-14.  Also, convert the receiver thermal noise to decibels relative to a Watt (dBW), and relative to a milliWatt (dBm).

\[
N := F_R \cdot k \cdot T_0 \cdot B_R = 7.9661 \times 10^{-15} \text{W}
\]

\[
= 10 \left( \frac{6}{10} \right) \cdot \left( 1.38 \times 10^{-23} \text{W} \cdot \text{sec} \cdot \text{K}^{-1} \right) \cdot (290 \text{K}) \cdot (500 \times 10^3 \text{Hz}) = 7.9661 \times 10^{-15} \text{W}
\]

\[
N_{\text{dBW}} := 10 \cdot \log \left( \frac{N}{1 \text{W}} \right) = -140.9875
\]

\[
10 \cdot \log \left( \frac{7.9661 \times 10^{-15} \text{W}}{1 \text{W}} \right) = -140.9875
\]

\[
N_{\text{dBm}} := N_{\text{dBW}} + 30 = -110.9875
\]

The equivalent noise temperature, $T_e$ (K), is given in Equation 2-20.  The noise power added by an actual receiver above that generated by an ideal receiver, $\Delta N$ (Watts), is given in Equation 2-18.  Also, convert the additional noise power to decibels relative to a Watt (dBW), and relative to a milliWatt (dBm).
\[ T_e := T_0 \cdot (F_R - 1) = 864.5108\, \text{K} \quad \Rightarrow \quad 290 \cdot \left( \frac{6}{10} \right) - 1 = 864.5108\, \text{K} \]

\[ \Delta N := k \cdot T_e \cdot B_R = 5.9651 \times 10^{-15}\, \text{W} \]

\[ = 1.38 \cdot 10^{-23} \cdot \frac{(\text{W} \cdot \text{sec})}{\text{K}} \cdot (864.5108\, \text{K}) \cdot (500 \cdot 10^3\, \text{Hz}) = 5.9651 \times 10^{-15}\, \text{W} \]

\[ \Delta N_{\text{dBW}} := 10 \cdot \log\left( \frac{\Delta N}{1\, \text{W}} \right) = -142.2438 \]

\[ 10 \cdot \log\left( \frac{5.9651 \times 10^{-15}\, \text{W}}{1\, \text{W}} \right) = -142.2438 \]

\[ \Delta N_{\text{dBm}} := \Delta N_{\text{dBW}} + 30 = -112.2438 \]

\[ = -142.24380488855604 + 30 = -112.2438 \]

This result clearly shows the additional noise is the dominate component of the receiver thermal noise power.
Exercise 2-4  The radar in Exercises 2 and 3 is transmitting at $f_c = 1$ GHz and has a signal processing gain $G_{sp} = 1$. What total radar-related losses, $L_R$ (no units and dB), is required to give a single pulse signal-to-noise ratio of unity ($SNR = 1$)?

We start by defining the values for all the terms and necessary constants.

\[
\begin{align*}
PR & \equiv 1 \times 10^6 \cdot W \\
G_{RT\_dBi} & \equiv 40 \\
f_c & \equiv 1 \times 10^9 \cdot Hz \\
R_{RT} & \equiv 500 \times 10^3 \cdot m \\
B_R & \equiv 500 \times 10^3 \cdot Hz \\
\sigma & \equiv 1 \cdot m^2 \\
F_{R\_dB} & \equiv 6 \\
T_0 & \equiv 290 \cdot K \\
k & \equiv 1.38 \times 10^{-23} \cdot \frac{W \cdot sec}{K} \\
c & \equiv 3 \times 10^8 \cdot \frac{m}{sec} \\
SNR & \equiv 1
\end{align*}
\]

Convert values from dB to absolute as we will be using algebraic equations.

\[
\begin{align*}
G_{RT} & = 10^{\left(\frac{G_{RT\_dBi}}{10}\right)} = 1 \times 10^4 \\
F_R & = 10^{\left(\frac{F_{R\_dB}}{10}\right)} = 3.9811
\end{align*}
\]

The single pulse signal-to-noise ratio (SNR) is given by Equation 2-25. Solve this equation for the total radar-related losses, $L_R$ (no units), as shown below:

\[
\begin{align*}
\text{SNR} & = \frac{PR \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma \cdot G_{sp}}{(4 \pi)^3 \cdot R_{RT}^4 \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R} \\
L_R & = \frac{PR \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma \cdot G_{sp}}{(4 \pi)^3 \cdot R_{RT}^4 \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot SNR}
\end{align*}
\]

We were given almost all the terms we need for this equation. We can compute the wavelength, $\lambda$ (m), using Equation 2-2.
\[
\lambda := \frac{c}{f_c} = 0.3 \text{m}
\]

\[
= \frac{3 \times 10^8 \frac{\text{m}}{\text{sec}}}{(1 \times 10^9 \text{Hz})} = 0.3 \text{m}
\]

We can now compute the total radar-related losses.

\[
L_R := \frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma \cdot G_{sp}}{(4\pi)^3 \cdot R_T^4 \cdot F_R \cdot k \cdot T_0 \cdot B \cdot \text{SNR}} = 9.1093
\]

\[
\left(1 \times 10^6 \cdot \text{W}\right) \cdot \left[\frac{40}{10}\right]^2 \cdot \left(\frac{3 \times 10^8 \cdot \text{m}}{1 \times 10^9 \cdot \text{Hz}}\right)^2 \cdot (1 \cdot \text{m}^2) \cdot (1) = 9.1093
\]

\[
(4\pi)^3 \cdot \left(500 \times 10^{-3} \cdot \text{m}\right)^4 \cdot \left[\frac{6}{10}\right]^4 \cdot \left(1.38 \times 10^{-23} \cdot \frac{\text{W} \cdot \text{sec}}{\text{K}}\right) \cdot (290 \cdot \text{K}) \cdot (500 \times 10^{-3} \cdot \text{Hz}) \cdot (1) = 9.1093
\]

\[
10 \log(L_R) = 9.5949 = 10 \cdot \log(9.1093169120822139) = 9.5949
\]
**Exercise 2-5** A radar system has the following characteristics: peak transmit power $P_R = 800$ kW, antenna gain in the direction of the target $G_{RT} = 38$ dBi, carrier frequency $f_c = 3$ GHz, signal processing gain $G_{sp} = 1$, receiver noise figure $F_R = 6$ dB, receiver bandwidth $B_R = 750$ kHz, total radar-related losses $L_R = 13$ dB, and radar transmit loss $L_{Rt} = 2$ dB. A target with a radar cross section $\sigma = 5$ m$^2$ is at a radar-to-target range $R_{RT} = 150$ km. Compute the following: a) the transmitted effective radiated power, $E_{RP}$ (Watts and dBW), b) the radar power density at the target, c) the power reflected off the target back to the radar, d) the received power density at the radar receive antenna, e) the received single pulse target signal power, $S$ (Watts and dBW), f) compare the received single pulse target signal power to the radar’s transmitted effective radiated power, g) the radar receiver thermal noise power, $N$ (Watts and dBW), and h) the single pulse target signal-to-noise ratio, $SNR$ (no units and dB).

We start by defining the values for all the terms and necessary constants.

- $P_R \equiv 800 \times 10^3 \text{W}$
- $G_{RT\_dBi} \equiv 38$
- $f_c \equiv 3 \times 10^9 \text{Hz}$
- $G_{sp} \equiv 1$
- $F_R\_dB \equiv 6$
- $B_R \equiv 750 \times 10^3 \text{Hz}$
- $L_R\_dB \equiv 13$
- $L_{Rt\_dB} \equiv 2$
- $\sigma \equiv 5 \text{m}^2$
- $R_{RT} \equiv 150 \times 10^3 \text{m}$
- $c \equiv 3 \times 10^8 \frac{\text{m}}{\text{sec}}$
- $T_0 \equiv 290 \text{K}$
- $k \equiv 1.38 \times 10^{-23} \frac{\text{W} \cdot \text{sec}}{\text{K}}$

Convert values from dB to absolute as we will be using algebraic equations.

- $G_{RT} := 10^{\left( \frac{G_{RT\_dBi}}{10} \right)} = 6.3096 \times 10^3$
- $F_R := 10^{\left( \frac{F_R\_dB}{10} \right)} = 3.9811$
- $L_R := 10^{\left( \frac{L_R\_dB}{10} \right)} = 19.9526$


\[
\begin{align*}
G_{RT} &= 10^{\left( \frac{38}{10} \right)} = 6.3096 \times 10^3 \\
F_R &= 10^{\left( \frac{6}{10} \right)} = 3.9811 \\
L_R &= 10^{\left( \frac{13}{10} \right)} = 19.9526
\end{align*}
\]

- $L_{Rt} := 10^{\left( \frac{L_{Rt\_dB}}{10} \right)} = 1.5849$

\[
\begin{align*}
L_{Rt} &= 10^{\left( \frac{2}{10} \right)} = 1.5849
\end{align*}
\]

a) Compute the transmitted effective radiated power (ERP) of the radar, $E_{RP}$ (Watts and dBW), using Equation 2-3.
ERPR := \frac{P_R \cdot G_{RT}}{L_{RT}} = 3.1849 \times 10^9 \text{ W}

\text{ERPR}_{\text{dBW}} := 10 \cdot \log \left( \frac{\text{ERPR}}{1 \text{ W}} \right) = 95.0309

b) Compute the radar power density (Watts / square meter) at the target using Equation 2-4.

\frac{P_R \cdot G_{RT}}{4 \pi \cdot R_{RT}^2 \cdot L_{RT}} = 0.0113 \cdot \frac{\text{W}}{\text{m}^2}

= \frac{800 \cdot 10^3 \cdot \text{W} \cdot \left( \frac{38}{10} \right)}{4 \pi \cdot \left( 150 \cdot 10^3 \text{ m} \right)^2} \cdot 10 \left( \frac{2}{10} \right) = 0.0113 \cdot \frac{\text{W}}{\text{m}^2}

\text{c) Compute the power reflected back to the radar due to the target RCS using Equation 2-5.}

\frac{P_R \cdot G_{RT} \cdot \sigma}{4 \pi \cdot R_{RT}^2 \cdot L_{RT}} = 0.0563 \text{ W}

= \frac{800 \cdot 10^3 \cdot \text{W} \cdot \left( \frac{38}{10} \right) \cdot \left( 5 \text{ m}^2 \right)}{4 \pi \cdot \left( 150 \cdot 10^3 \text{ m} \right)^2} \cdot 10 \left( \frac{2}{10} \right) = 0.0563 \text{ W}

d) Compute the received power density at the radar receive antenna using Equation 2-6.
\[
\frac{P_R \cdot G_{RT} \cdot \sigma}{(4\pi)^2 \cdot R_{RT}^4 \cdot L_R} = 1.9919 \times 10^{-13} \text{ W/m}^2
\]

\[
= \frac{800 \cdot 10^3 \text{ W} \cdot 10 \left(\frac{38}{10}\right) \cdot (5 \text{ m}^2)}{(4 \cdot \pi)^2 \left(150 \cdot 10^3 \text{ m} \right)^4 \cdot 10^{\left(\frac{2}{10}\right)}} = 1.9919 \times 10^{-13} \text{ W/m}^2
\]

e) Compute the received single pulse target signal power, \(S\) (Watts) and \(S_{\text{dBW}}\) (dBW), using Equation 2-11. Since we will need the wavelength, \(\lambda\) (meters), we will compute it first using Equation 2-2.

\[
\lambda = \frac{c}{f_c} = 0.1 \text{ m}
\]

\[
S := \frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma}{(4\pi)^3 \cdot R_{RT}^4 \cdot L_R} = 7.9445 \times 10^{-14} \text{ W}
\]

\[
= \frac{800 \cdot 10^3 \text{ W} \cdot \left[10^{\left(\frac{38}{10}\right)}\right]^2 \cdot 3 \cdot 10^8 \left(\frac{\text{m}}{\text{sec}}\right) \cdot (5 \text{ m}^2)}{(4 \cdot \pi)^3 \cdot \left(150 \cdot 10^3 \text{ m} \right)^4 \cdot 10^{\left(\frac{13}{10}\right)}} = 7.9445 \times 10^{-14} \text{ W}
\]

\[
S_{\text{dBW}} := 10 \log \left(\frac{S}{1\text{ W}}\right) = -130.9993
\]

\[
10 \log \left(\frac{7.9445 \times 10^{-14} \text{ W}}{1\text{ W}}\right) = -130.9993
\]

f) Compare the received target signal power to the radar's transmitted effective radiated power. The radar system received approximately 23
orders of magnitude (power of 10) less power than it transmitted. The vast majority of the power is lost by the propagation out to, and back from the target - here at 300 km total propagation path length.

\[
\frac{S}{\text{ERPR}} = 2.4945 \times 10^{-23}
\]

\[
10 \log \left( \frac{S}{\text{ERPR}} \right) = -226.0302
\]

\[
\frac{S}{\text{ERPR}} \approx 7.9445 \times 10^{-14} \, \text{W} = 2.4944 \times 10^{-23} \, \text{W}
\]

\[
\frac{S}{\text{ERPR}} \approx 3.1849 \times 10^9 \, \text{W}
\]

\[
10 \log \left( \frac{7.9445 \times 10^{-14} \, \text{W}}{3.1849 \times 10^9 \, \text{W}} \right) = -226.0303
\]

**g)** Compute the radar receiver thermal noise power, \( N \) (Watts) and \( N_{\text{dBW}} \) (dBW), using Equation 2-14.

\[
N := F_R \cdot k \cdot T_0 \cdot B_R = 1.1949 \times 10^{-14} \, \text{W}
\]

\[
= 10^{\left( \frac{6}{10} \right)} \cdot 1.38 \cdot 10^{-23} \cdot \left( \frac{\text{W} \cdot \text{sec}}{\text{K}} \right) \cdot (290 \, \text{K}) \cdot (750 \cdot 10^3 \, \text{Hz}) = 1.1949 \times 10^{-14} \, \text{W}
\]

\[
N_{\text{dBW}} := 10 \log \left( \frac{N}{1 \, \text{W}} \right) = -139.2266
\]

\[
10 \log \left( \frac{1.1949 \times 10^{-14} \, \text{W}}{1 \, \text{W}} \right) = -139.2267
\]

**h)** Compute the single pulse target signal-to-noise ratio, \( \text{SNR} \) (no units) and \( \text{SNR}_{\text{dB}} \) (dB), using Equation 2-25.

\[
\text{SNR} := \frac{P_R \cdot G_{RT}^2 \cdot \chi^2 \cdot \sigma \cdot G_{sp}}{(4 \pi)^3 \cdot R_{RT}^4 \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R} = 6.6486
\]
\[
\left(800 \times 10^{-3} \cdot \text{W}\right) \cdot \left[10 \left(\frac{38}{10}\right)\right]^2 \cdot \left(\frac{3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}}{3 \times 10^9 \cdot \text{Hz}}\right)^2 \cdot (5 \cdot \text{m}^2) \cdot (1)\]

\[
(4\pi)^3 \cdot \left(150 \times 10^{-3} \cdot \text{m}\right)^4 \cdot 10 \left(\frac{6}{10}\right) \cdot \left(1.38 \times 10^{-23} \cdot \frac{\text{W} \cdot \text{sec}}{\text{K}}\right) \cdot (290 \cdot \text{K}) \cdot (750 \times 10^{-3} \cdot \text{Hz}) \cdot 10 \left(\frac{13}{10}\right) = 6.6486
\]

SNR_{dB} := 10 \log (\text{SNR}) = 8.2273

\[
= 10 \cdot \log (6.6485511659026288) = 8.2273
\]
Exercise 2-6  The Anti-Ballistic Missile Defense Treaty of the Strategic Arms Limitations accords with Russia limits radars at ABM sites in the two countries to $3 \times 10^6$ Watt-meters-squared of power-aperture (PA) product. The radar designers want to use a peak transmit power $P_R = 250 \text{ kW}$ operating at a frequency $f_c = 6 \text{ GHz}$. Assuming such a radar requires a signal-to-noise ratio for detection $\text{SNR}_{dt} = 13 \text{ dB}$, has a receiver bandwidth $B_R = 300 \text{ kHz}$, receiver noise figure $F_R = 5 \text{ dB}$, signal processing gain $G_{sp} = 1$, and total radar related losses $L_R = 8 \text{ dB}$. What would be its maximum possible detection range against a target with a radar cross section $\sigma = 10 \text{ m}^2$ target?

We start by defining the values for all the terms and necessary constants.

$$
\begin{align*}
\text{PA} & \equiv 3 \times 10^6 \cdot \text{W} \cdot \text{m}^2 \\
\text{f}_c & \equiv 6 \times 10^9 \cdot \text{Hz} \\
B_R & \equiv 300 \times 10^3 \cdot \text{Hz} \\
P_R & \equiv 250 \times 10^3 \cdot \text{W} \\
\text{SNR}_{dt \_dB} & \equiv 13 \\
F_R \_{dB} & \equiv 5 \\
G_{sp} & \equiv 1 \\
L_R \_{dB} & \equiv 8 \\
\sigma & \equiv 10 \cdot \text{m}^2 \\
T_0 & \equiv 290 \cdot \text{K} \\
c & \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}} \\
k & \equiv 1.38 \times 10^{-23} \cdot \frac{\text{W} \cdot \text{sec}}{\text{K}} \\
\end{align*}
$$

The single pulse radar detection range, $R_{dt}$ (meters), is given by Equation 2-26.

$$
R_{dt} = \frac{4 \cdot P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp}}{(4 \pi)^3 \cdot \text{SNR}_{dt} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}
$$

We were given almost all the values needed my this equation. The first step is to solve for the maximum available aperture size, $A$ (m$^2$). We know the power-aperture (PA) product limit and the peak transmitter power, thus we can solve for the aperture size as shown below:

$$
A := \frac{\text{PA}}{P_R} = 12 \text{ m}^2
$$

Next we compute the maximum possible antenna gain we can obtain from this aperture size. We will assume the antenna efficiency is 100% because we are after the maximum possible detection range, which will we will get with the maximum possible antenna gain.
Equation 2-8 is used to compute the antenna gain. Since we will need the wavelength we will compute it here as well using Equation 2-2.

\[
\lambda := \frac{c}{f_c} = 0.05\text{m}
\]

\[
G := \frac{4\pi \cdot A}{\lambda^2} = 6.0319 \times 10^4
\]

\[
G_{RT} := G = 6.0319 \times 10^4
\]

We can now solve the for the detection range, \(R_{dt}\) (m). But first since we are using an algebraic equation we will convert the detection threshold, receiver noise figure, and total radar related losses from decibels to absolute.

\[
\frac{\text{SNR}_{dt_{\text{db}}}}{10} = 19.9526
\]

\[
\frac{F_R_{\text{db}}}{10} = 3.1623
\]

\[
\frac{L_R_{\text{db}}}{10} = 6.3096
\]

\[
R_{dt} = 393.4944\text{ km}
\]
Just to get an idea of effective radiated power (ERP) we will compute the radar ERP, $\text{ERP}_R$ (Watts), using Equation 2-3. We will assume the radar transmit loss $L_{Rt} = 1$ (0 dB).

$$\text{ERP}_R := \frac{P_{R,GRT}}{1} = 1.508 \times 10^{10} \text{W}$$
**Exercise 2-7**  The cost of a radar is the cost of power plus the cost of aperture plus a constant. The cost of power is the cost/kiloWatt multiplied by the number of kilowatts: $C_P = C_{kW} \cdot N_{kW}$. And the cost of aperture is the cost/square meter multiplied by the number of square meters: $C_A = C_{m2} \cdot N_{m2}$. Show for the minimum cost $2C_P = C_A$. (Hint: Assign a design power-aperture product.)

We start by defining the cost of the radar, $C_R$, as the sum of the cost of power, cost of aperture, and a constant for all other components, $C_{other}$.

$$C_R = C_P + C_A + C_{other} = C_{kW} \cdot N_{kW} + C_{m2} \cdot N_{m2} + C_{other}$$

The S/N, Equation 2-25, relates antenna gain, and thus aperture size, $A$ (the same as $N_{m2}$ in this exercise), and power $P$ (the same as $N_{kW}$ in this exercise) so that we can make the cost equation with only the parameter $A$. We will also need the relationship between antenna gain, $G$, and aperture size, $A$ (Equation 2-8).

$$\frac{S}{N} = \frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp}}{(4\pi)^3 \cdot R_{RT} \cdot 4 \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R} \quad \quad A = \frac{G \cdot \lambda^2}{4\pi} \quad \quad G = \frac{4\pi \cdot A}{\lambda^2}$$

Now we do a little algebra to solve the S/N equation in terms of the power-aperture product.

$$\frac{S}{N} = \frac{P_R \cdot \left(\frac{4\pi \cdot A}{\lambda^2}\right)^2 \cdot \lambda^2 \cdot \sigma \cdot G_{sp}}{(4\pi)^3 \cdot R_{RT} \cdot 4 \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R} = \frac{P_R \cdot A^2 \cdot \sigma \cdot G_{sp}}{(4\pi)^3 \cdot \lambda^2 \cdot R_{RT} \cdot 4 \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}$$

$$P_R \cdot A^2 = \frac{(4\pi)^3 \cdot \lambda^2 \cdot R_{RT} \cdot 4 \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R \cdot \left(\frac{S}{N}\right)}{\sigma \cdot G_{sp}}$$

We will represent all the terms to the right of the equal sign with the constant $X$. This is us using the hint from above.
\[ P_R \cdot A^2 = X \]

\[ X = \frac{(4\pi) \cdot \lambda^2 \cdot R_{RT}^4 \cdot P_R \cdot k \cdot T_0 \cdot B_R \cdot L_R \cdot \left( \frac{S}{N} \right)}{\sigma \cdot G_{SP}} \]

We can now solve for power as a function of the area. We now will use the power and area terms in the radar cost equation.

\[ P_R = \frac{X}{A^2} \]

\[ C_R = C_{kW} \cdot \left( \frac{X}{A^2} \right) + C_{m2} \cdot (A) + C_{other} \]

We find the minimum of the cost equation by differentiating \( C_R \) with respect to \( A \), and setting the result equal to zero.

\[ \frac{d}{dA} C_R = \frac{-2C_{kW} \cdot X}{A^3} + C_{m2} = 0 \]

\[ C_{m2} = \frac{2C_{kW} \cdot X}{A^3} \]

Substituting the term \( X \) back into this equation and rearranging so we collect the aperture terms on one side of equation and the power terms on the other gives us the following.

\[ X = P_R \cdot A^2 \]

\[ C_{m2} = \frac{2C_{kW} \cdot P_R \cdot A^2}{A^3} = \frac{2C_{kW} \cdot P_R}{A} \]

\[ C_{m2} \cdot A = 2C_{kW} \cdot P_R \]

Now we substitute the aperture and power cost terms, \( C_A \) and \( C_P \), back into the equation and we are left with the following relationship.

\[ C_A = 2C_P \]
Exercise 3-1  A radar system has the following characteristics: time to make an individual detection decision \( T_d = 25 \text{ msec} \), number of range gates \( n_{rg} = 300 \), number of Doppler filters \( n_{df} = 125 \), and number of antenna beam positions \( n_b = 640 \). The radar system is required to have an average time between false alarms \( T_{fa} = 60 \text{ sec} \). Compute the number of detection decisions, \( N_d \), the number of detection decisions per second, the probability of false alarm, \( P_{fa} \), the average false alarm rate, FAR, the number of false alarms in one time period of interest, \( N_{fa} \), and signal-to-noise ratio threshold, SNR (no units and dB).

We start by defining the values for all the terms.

\[
\begin{align*}
T_d &\equiv 25 \times 10^{-3} \text{ sec} \\
n_{rg} &\equiv 300 \\
n_{df} &\equiv 125 \\
n_b &\equiv 640 \\
T_{fa} &\equiv 60 \text{ sec}
\end{align*}
\]

The number of detection decisions, \( N_d \) (no units), is given by Equation (3-11).

\[
N_d := n_{rg} \cdot n_{df} \cdot n_b = 2.4 \times 10^7
\]

The number of detection decisions per second is given by Equation (3-13).

\[
\frac{n_{rg} \cdot n_{df} \cdot n_b}{T_d} = 9.6 \times 10^8 \frac{1}{s}
\]

The probability of false alarm, \( P_{fa} \) (no units), is given by Equation (3-16).

\[
P_{fa} := \frac{T_d}{T_{fa} \cdot N_d} = 1.7361 \times 10^{-11}
\]

The false alarm rate, FAR (false alarm per sec), is given by Equation (3-14).
The number of false alarms in one time period of interest, \( N_{fa} \) (no units), is given by Equation (3-12).

\[
N_{fa} := N_d \cdot P_{fa} = 4.1667 \times 10^{-4}
\]

\[
= 24000000 \cdot (1.7361 \cdot 10^{-11}) = 4.166 \times 10^{-4}
\]

The signal-to-noise ratio threshold, \( SNR \) (no units) and \( SNR_{dB} \) (dB), is given by Equation (3-9).

\[
SNR := -\ln(P_{fa}) = 24.7768
\]

\[
= -\ln(1.736111111111114E-11) = 24.7768
\]

\[
SNR_{dB} := 10 \log (SNR) = 13.9405
\]

\[
= 10 \cdot \log (24.776788404648258) = 13.9405
\]
**Exercise 3-2** Use the detection theory equations, Equations (3-20) and (3-22), to calculate the signal-to-noise ratio required for detection (detection threshold), \( D_0 \) (dB), to provide a single pulse probability of detection \( P_d = 0.9 \) and probability of false alarm \( P_{fa} = 10^{-6} \). Compare with Figure 3-9, Table 3.1, and Albersheim's formula, Equations (3-25) and (3-26).

We start by defining the values for all the terms.

\[
P_d = 0.9 \quad P_{fa} = 1 \times 10^{-6}
\]

Equation 3-20 relates probability of detection, probability of false alarm, and \( S/N \) required for detection (detection threshold), \( D_0 \) (absolute). Unfortunately, it is \( P_d \) as a function of \( D_0 \) and \( P_{fa} \). We need to try a few \( D_0 \) values in the equation below, and see what \( P_d \) we get. A few \( D_0 \) values are shown before we narrow in on the actual value.

\[
\begin{align*}
D_{0_1} & := 15 & 10 \log (D_{0_1}) & = 11.7609 \\
P_{d_1} & := 0.5 \cdot \text{erfc} \left( \sqrt{-\ln(P_{fa})} \right) - \sqrt{D_{0_1} + 0.5} & = 0.6222 \\
D_{0_2} & := 25 & 10 \log (D_{0_2}) & = 13.9794 \\
P_{d_2} & := 0.5 \cdot \text{erfc} \left( \sqrt{-\ln(P_{fa})} \right) - \sqrt{D_{0_2} + 0.5} & = 0.9703 \\
D_{0_3} & := 20.9 & 10 \log (D_{0_3}) & = 13.2015 \\
P_{d_3} & := 0.5 \cdot \text{erfc} \left( \sqrt{-\ln(P_{fa})} \right) - \sqrt{D_{0_3} + 0.5} & = 0.9007
\end{align*}
\]

The \( S/N \) from Figure 3.9 is approximately 13.2 dB, the \( S/N \) from Table 3-1 is also 13.2 dB.

The signal-to-noise ratio required for detection (detection threshold), \( D_0 \) (absolute), is given by Equation 3-22 as shown below. Note: "qnorm" using \((1-P_d)\) is the inverse Q probability function in Mathcad.

\[
\left( \sqrt{-\ln(P_{fa})} - \frac{1}{\sqrt{2}} \cdot \text{qnorm}(1-P_d, 0, 1) \right)^2 - 0.5 = 20.8732 \quad 10 \log (20.8732) = 13.1959
\]
The comparison with Albersheim's formula, Equations 3-25 and 3-26, is given below.

\[
A := \ln \left( \frac{0.62}{P_{fa}} \right) = 13.3375 \\
= \ln \left( \frac{0.62}{1 \times 10^{-6}} \right) = 13.3375
\]

\[
B := \ln \left( \frac{P_d}{1 - P_d} \right) = 2.1972 \\
= \ln \left( \frac{0.9}{1 - 0.9} \right) = 2.1972
\]

\[
10 \log (A + 0.12 \cdot A \cdot B + 1.7 \cdot B) = 13.1364
\]

\[
10 \log [13.3375 + (0.12) \cdot (13.3375) \cdot (2.1972) + (1.7) \cdot (2.1972)] = 13.1364
\]
**Exercise 3-3** A radar system requires the following detection criteria $P_d = 0.5$ and $P_{fa} = 10^{-6}$. What is the signal-to-noise ratio required for detection (detection threshold) for Swerling Case 0, 1, 2, 3, and 4 targets? Compute the results using the equations in Section 3.2.4 and then compare them to the results obtained in Figure 3-10 and Tables 3-1, 3-3, and 3-4. If the detection range is 100 km when the threshold is set based on a Swerling Case 0 target, what is the detection range for the other Swerling Cases?

We start by defining the values for all the terms.

$$P_d = 0.5, \quad P_{fa} = 10^{-6}, \quad R_{dt_{SW0}} = 100 \times 10^3 \text{ m}$$

**Note:** this solution uses North's equations using the Q probability integral (qnorm in Mathcad). The solution in Exercise 3-3-1 uses Albersheim's equation. As we will see the results from both equations are almost identical.

We will start by solving for the detection threshold for a Swerling 0 target, $D_0$ (no units) and $D_{0\_dB}$ (dB), using Equation (3-22).

$$D_0 := \left( \sqrt{-\ln(P_{fa})} - \frac{1}{\sqrt{2}} \cdot \text{qnorm}(1 - P_d, 0, 1) \right)^2 - 0.5 = 13.3155$$

$$D_{0\_dB} := 10 \log(D_0) = 11.2436 \quad \Rightarrow \quad 10 \log(13.315510557964274) = 11.2436$$

We can read the same value from Figure 3-9 and Table 3-1.

The detection threshold for a Swerling 1 target, $D_1$ (no units) and $D_{1\_dB}$ (dB), is given in Equation (3-28). The detection threshold for a Swerling 2 target, $D_2$ (no units) and $D_{2\_dB}$ (dB), is the same as that for a Swerling 1 target (see Equations (3-28) thru (3-30)).

$$D_1 := \frac{\ln(P_{fa})}{\ln(P_d)} - 1 = 18.9316$$

$$D_{1\_dB} := 10 \log(D_1) = 12.7719 \quad \Rightarrow \quad 10 \log(18.931568569324174) = 12.7719$$
\[ D_2 := D_1 = 18.9316 \]
\[ D_{2\_dB} := 10 \log(D_2) = 12.7719 \]

We determine the detection threshold for Swerling 3 and 4 targets by first computing the additional signal-to-noise ratio relative to a Swerling 0 target required for a Swerling 1 target, \( \Delta \text{SNR}_{f1} \) (no units) and \( \Delta \text{SNR}_{f1\_dB} \) (dB), using Equation (3-29). We then compute the additional signal-to-noise ratio relative to a Swerling 0 target required for a Swerling 3 target, \( \Delta \text{SNR}_{f3} \) (no units) and \( \Delta \text{SNR}_{f3\_dB} \) (dB), using Equation (3-30).

\[
\Delta \text{SNR}_{f1} := \frac{D_1}{D_0} = 1.4218
\]
\[
\Delta \text{SNR}_{f1\_dB} := 10 \log(\Delta \text{SNR}_{f1}) = 1.5283
\]
\[
\Delta \text{SNR}_{f3} := \Delta \text{SNR}_{f1}^2 = 1.1924
\]
\[
\Delta \text{SNR}_{f3\_dB} := 10 \log(\Delta \text{SNR}_{f3}) = 0.7641
\]

We can read the same values from Figure 3-10, Table 3-3, and Table 3-4.

We now compute the detection threshold for a Swerling 3 target, \( D_3 \) (no units) and \( D_{3\_dB} \) (dB). The detection threshold for a Swerling 4 target, \( D_4 \) (no units) and \( D_{4\_dB} \) (dB), is the same as that for a Swerling 3 target (see Equations see Equations (3-28) thru (3-30)).

\[
D_3 := D_0 \cdot \Delta \text{SNR}_{f3} = 15.8771
\]
\[
D_{3\_dB} := D_{0\_dB} + \Delta \text{SNR}_{f3\_dB} = 12.0077
\]
\[
D_4 := D_3 = 15.8771 \quad D_{4\_dB} := 10 \log(D_4) = 12.0077
\]

We use Equation (3-58) to compute new detection range based a new detection threshold and old detection range. First for a Swerling 1 or 2 target.
relative to a Swerling 0 target, $R_{dt_{SW1}}$ (meters), and then for a Swerling 3 or 4 target relative to a Swerling 0 target, $R_{dt_{SW3}}$ (meters).

\[
R_{dt2} = \frac{4}{\sqrt{SNR_{dt2}}} \cdot R_{dt1}
\]

\[
R_{dt_{SW1}} := \sqrt{\frac{D_0}{D_1}} \cdot R_{dt_{SW0}} = 9.1578 \times 10^4 \text{m}
\]

\[
R_{dt_{SW1}} = 91.5783 \text{ km}
\]

\[
R_{dt_{SW3}} := \sqrt{\frac{D_0}{D_3}} \cdot R_{dt_{SW0}} = 9.5697 \times 10^4 \text{m}
\]

\[
R_{dt_{SW3}} = 95.6966 \text{ km}
\]

As expected; since the detection thresholds for the Swerling 1, 2, 3, and 4 targets is higher than for a Swerling 0 target, the detection ranges for the other Swerling cases are less than for a Swerling 0 target.
**Exercise 3-3-1** A radar system requires the following detection criteria \( P_d = 0.5 \) and \( P_{fa} = 10^{-6} \). What is the signal-to-noise ratio required for detection (detection threshold) for Swerling Case 0, 1, 2, 3, and 4 targets? Compute the results using the equations in Section 3.2.4 and then compare them to the results obtained in Figure 3-10 and Tables 3-1, 3-3, and 3-4. If the detection range is 100 km when the threshold is set based on a Swerling Case 0 target, what is the detection range for the other Swerling Cases?

We start by defining the values for all the terms.

\[
P_d = 0.5 \quad P_{fa} = 10^{-6} \quad R_{dt\_sw0} = 100 \times 10^3 \cdot m
\]

**Note:** this solution uses Albersheim's equation. The solution in Exercise 3-3 uses North's equations using the Q probability integral. As we will see the results from both equations are almost identical.

We will start by solving for the detection threshold for a Swerling 0 target, \( D_0 \) (no units) and \( D_0\_db \) (dB), using Equations (3-25) and (3-26).

\[
A := \ln \left( \frac{0.62}{P_{fa}} \right) = 13.3375
\]

\[
B := \ln \left( \frac{P_d}{1 - P_d} \right) = 0
\]

\[
D_0\_db := 10 \log (A + 0.12 \cdot A \cdot B + 1.7 \cdot B) = 11.2507
\]

\[
D_0 := 10 \left( \frac{D_0\_db}{10} \right) = 13.3375
\]

We can read the same value from Figure 3-9 and Table 3-1.

The detection threshold for a Swerling 1 target, \( D_1 \) (no units) and \( D_1\_db \) (dB), is given in Equation (3-28). The detection threshold for a Swerling 2 target, \( D_2 \) (no units) and \( D_2\_db \) (dB), is the same as that for a Swerling 1 target (see Equations (3-28) thru (3-30)).
We determine the detection threshold for Swerling 3 and 4 targets by first computing the additional signal-to-noise ratio relative to a Swerling 0 target required for a Swerling 1 target, $\Delta SNR_{f1}$ (no units) and $\Delta SNR_{f1_{dB}}$ (dB), using Equation (3-29). We then compute the additional signal-to-noise ratio relative to a Swerling 0 target required for a Swerling 3 target, $\Delta SNR_{f3}$ (no units) and $\Delta SNR_{f3_{dB}}$ (dB), using Equation (3-30).

\[
\Delta SNR_{f1} := \frac{D_1}{D_0} = 1.4194
\]
\[
\Delta SNR_{f1_{dB}} := 10 \log \left( \Delta SNR_{f1} \right) = 1.5211
\]
\[
\Delta SNR_{f3} := \frac{1}{\Delta SNR_{f1}^2} = 1.1914
\]
\[
\Delta SNR_{f3_{dB}} := 10 \log \left( \Delta SNR_{f3} \right) = 0.7606
\]
\[
D_3 := D_0 \cdot \Delta SNR_{f3} = 15.8902
\]
\[
D_{3_{dB}} := D_0_{dB} + \Delta SNR_{f3_{dB}} = 12.0113
\]
\[
D_4 := D_3 = 15.8902
\]
\[
D_{4_{dB}} := 10 \log \left( D_4 \right) = 12.0113
\]
\[
R_{dt_2} = \sqrt[4]{\frac{\text{SNR}_{dt_1}}{\text{SNR}_{dt_2}}} \cdot R_{dt_1}
\]

\[
R_{dt_{sw1}} := \frac{4D_0}{D_1} \cdot R_{dt_{sw0}} = 9.1616 \times 10^4 \text{ m}
\]

\[
R_{dt_{sw1}} = 91.6161 \text{ km}
\]

\[
R_{dt_{sw3}} := \frac{4D_0}{D_3} \cdot R_{dt_{sw0}} = 9.5716 \times 10^4 \text{ m}
\]

\[
R_{dt_{sw3}} = 95.7163 \text{ km}
\]

As expected; since the detection thresholds for the Swerling 1, 2, 3, and 4 targets is higher than for a Swerling 0 target, the detection ranges for the other Swerling cases are less than for a Swerling 0 target.
Exercise 3-4  A radar system requires the following detection criteria \( P_d = 0.9 \) and \( P_{fa} = 10^{-8} \). What is the signal-to-noise ratio required for detection (detection threshold) for Swerling Case 0, 1, 2, 3, and 4 targets? Compute the results using the equations in Section 3.2.4 and then compare them to the results obtained in Figure 3-10 and Tables 3-1, 3-3, and 3-4. If the detection range is 100 km when the threshold is set based on a Swerling Case 0 target, what is the detection range for the other Swerling Cases?

We start by defining the values for all the terms.

\[
P_d = 0.9 \quad P_{fa} = 10^{-8} \quad R_{dt,SW0} = 100 \times 10^3 \cdot m
\]

Note: this solution uses North's equations using the Q probability integral (qnorm in Mathcad). The solution in Exercise 3-4-1 uses Albersheim's equation. As we will see the results from both equations are almost identical.

We will start by solving for the detection threshold for a Swerling 0 target, \( D_0 \) (no units) and \( D_{0\_dB} \) (dB), using Equation (3-22).

\[
D_0 := \left( \sqrt{-\ln(P_{fa})} - \frac{1}{\sqrt{2}} \cdot \text{qnorm}(1 - P_d, 0, 1) \right)^2 - 0.5 = 26.5205
\]

\[
D_{0\_dB} := 10 \log(D_0) = 14.2358
\]

We can read the same value from Figure 3-9 and Table 3-1.

The detection threshold for a Swerling 1 target, \( D_1 \) (no units) and \( D_{1\_dB} \) (dB), is given in Equation (3-28). The detection threshold for a Swerling 2 target, \( D_2 \) (no units) and \( D_{2\_dB} \) (dB), is the same as that for a Swerling 1 target (see Equations (3-28) thru (3-30)).
D₂ := D₁ = 173.8348 
D₂_dB := 10 \log(D₂) = 22.4014

We determine the detection threshold for Swerling 3 and 4 targets by first computing the additional signal-to-noise ratio relative to a Swerling 0 target required for a Swerling 1 target, \( \Delta \text{SNR}_{f₁} \) (no units) and \( \Delta \text{SNR}_{f₁ \_dB} \) (dB), using Equation (3-29). We then compute the additional signal-to-noise ratio relative to a Swerling 0 target required for a Swerling 3 target, \( \Delta \text{SNR}_{f₃} \) (no units) and \( \Delta \text{SNR}_{f₃ \_dB} \) (dB), using Equation (3-30).

\[ \Delta \text{SNR}_{f₁} := \frac{D₁}{D₀} = 6.5547 \]
\[ \Delta \text{SNR}_{f₁ \_dB} := 10 \log(\Delta \text{SNR}_{f₁}) = 8.1655 \]

\[ \Delta \text{SNR}_{f₃} := \Delta \text{SNR}_{f₁} \left( \frac{1}{2} \right)^2 = 2.5602 \]
\[ \Delta \text{SNR}_{f₃ \_dB} := 10 \log(\Delta \text{SNR}_{f₃}) = 4.0828 \]

\[ D₃ := D₀ \cdot \Delta \text{SNR}_{f₃} = 67.8984 \]
\[ D₃ \_dB := D₀ \_dB + \Delta \text{SNR}_{f₃ \_dB} = 18.3186 \]

\[ D₄ := D₃ = 67.8984 \quad D₄ \_dB := 10 \log(D₄) = 18.3186 \]

We use Equation (3-58) to compute new detection range based a new detection threshold and old detection range. First for a Swerling 1 or 2 target relative to a Swerling 0 target, \( R_{dt\_SW₁} \) (meters), and then for a Swerling 3 or 4 target relative to a Swerling 0 target, \( R_{dt\_SW₃} \) (meters).
\[ R_{dt2} = 4 \sqrt{\frac{SNR_{dt1}}{SNR_{dt2}}} \cdot R_{dt1} \]

\[ R_{dt_{Sw1}} := 4 \sqrt{\frac{D_0}{D_1}} \cdot R_{dt_{Sw0}} = 6.2497 \times 10^4 \text{ m} \]

\[ R_{dt_{Sw1}} = 62.4973 \text{ km} \]

\[ R_{dt_{Sw3}} := 4 \sqrt{\frac{D_0}{D_3}} \cdot R_{dt_{Sw0}} = 7.9055 \times 10^4 \text{ m} \]

\[ R_{dt_{Sw3}} = 79.0552 \text{ km} \]

As expected; since the detection thresholds for the Swerling 1, 2, 3, and 4 targets is higher than for a Swerling 0 target, the detection ranges for the other Swerling cases are less than for a Swerling 0 target.
**Exercise 3-4-1** A radar system requires the following detection criteria \( P_d = 0.9 \) and \( P_{fa} = 10^{-8} \). What is the signal-to-noise ratio required for detection (detection threshold) for Swerling Case 0, 1, 2, 3, and 4 targets? Compute the results using the equations in Section 3.2.4 and then compare them to the results obtained in Figure 3-10 and Tables 3-1, 3-3, and 3-4. If the detection range is 100 km when the threshold is set based on a Swerling Case 0 target, what is the detection range for the other Swerling Cases?

We start by defining the values for all the terms.

\[
P_d = 0.9 \quad P_{fa} = 10^{-8} \quad R_{dt_SW0} = 100 \times 10^3 \cdot m
\]

*Note:* this solution uses Albersheim's equation. The solution in Exercise 3-4 uses North's equations using the Q probability integral. As we will see the results from both equations are almost identical.

We will start by solving for the detection threshold for a Swerling 0 target, \( D_0 \) (no units) and \( D_{0\_dB} \) (dB), using Equations (3-25) and (3-26).

\[
A := \ln \left( \frac{0.62}{P_{fa}} \right) = 17.9426
\]

\[
B := \ln \left( \frac{P_d}{1 - P_d} \right) = 2.1972
\]

\[
D_{0\_dB} := 10 \log (A + 0.12 \cdot A \cdot B + 1.7 \cdot B) = 14.2175
\]

\[
10 \log \left[ 17.9426 + (0.12) \cdot (17.9246) \cdot (2.1972) + (1.7) \cdot (2.1972) \right] = 14.2167
\]

\[
D_0 := \left( \frac{D_{0\_dB}}{10} \right) = 26.4088
\]

\[
= 10 \left( \frac{14.217488185180914}{10} \right) = 26.4088
\]

We can read the same value from Figure 3-9 and Table 3-1.

The detection threshold for a Swerling 1 target, \( D_1 \) (no units) and \( D_{1\_dB} \) (dB), is given in Equation (3-28). The detection threshold for a Swerling 2 target, \( D_2 \) (no units) and \( D_{2\_dB} \) (dB), is the same as that for a Swerling 1 target (see Equations (3-28) thru (3-30)).
\[ D_1 := \frac{\ln(P_{fa})}{\ln(P_d)} - 1 = 173.8348 \]

\[ \ln P_{fa} = \frac{\ln(10^{-8})}{\ln(0.9)} - 1 = 173.8348 \]

\[ D_{1_{\text{dB}}} := 10 \log(D_1) = 22.4014 \]

\[ = 10 \cdot \log(173.8347626142627) = 22.4014 \]

\[ D_2 := D_1 = 173.8348 \quad \text{and} \quad D_{2_{\text{dB}}} := 10 \log(D_2) = 22.4014 \]

We determine the detection threshold for Swerling 3 and 4 targets by first computing the additional signal-to-noise ratio relative to a Swerling 0 target required for a Swerling 1 target, \( \Delta \text{SNR}_{f1} \) (no units) and \( \Delta \text{SNR}_{f1_{\text{dB}}} \) (dB), using Equation (3-29). We then compute the additional signal-to-noise ratio relative to a Swerling 0 target required for a Swerling 3 target, \( \Delta \text{SNR}_{f3} \) (no units) and \( \Delta \text{SNR}_{f3_{\text{dB}}} \) (dB), using Equation (3-30).

\[ \Delta \text{SNR}_{f1} := \frac{D_1}{D_0} = 6.5825 \]

\[ = \frac{173.8347626142627}{26.408809178624661} = 6.5825 \]

\[ \Delta \text{SNR}_{f1_{\text{dB}}} := 10 \log(\Delta \text{SNR}_{f1}) = 8.1839 \]

\[ = 10 \cdot \log(6.5824536592496141) = 8.1839 \]

\[ \Delta \text{SNR}_{f3} := \Delta \text{SNR}_{f1}^2 = 2.5656 \]

\[ = \frac{6.5824536592496141}{2} = 2.5656 \]

\[ \Delta \text{SNR}_{f3_{\text{dB}}} := 10 \log(\Delta \text{SNR}_{f3}) = 4.0919 \]

\[ = 10 \cdot \log(2.56562929108038) = 4.0919 \]

\[ D_3 := D_0 \cdot \Delta \text{SNR}_{f3} = 67.7552 \]

\[ = 26.408809178624661 \cdot 2.56562929108038 = 67.7552 \]

\[ D_{3_{\text{dB}}} := D_{0_{\text{dB}}} + \Delta \text{SNR}_{f3_{\text{dB}}} = 18.3094 \]

\[ = 14.217488185180914 + 4.091939051687322 = 18.3094 \]

\[ D_4 := D_3 = 67.7552 \quad \text{and} \quad D_{4_{\text{dB}}} := 10 \log(D_4) = 18.3094 \]
We use Equation (3-58) to compute new detection range based a new detection threshold and old detection range. First for a Swerling 1 or 2 target relative to a Swerling 0 target, \( R_{dt\_SW1} \) (meters), and then for a Swerling 3 or 4 target relative to a Swerling 0 target, \( R_{dt\_SW3} \) (meters).

\[
R_{dt2} = \sqrt[4]{\frac{\text{SNR}_{dt1}}{\text{SNR}_{dt2}}} \cdot R_{dt1}
\]

\[
R_{dt\_SW1} := \sqrt[4]{\frac{D_0}{D_1}} \cdot R_{dt\_SW0} = 6.2431 \times 10^4 \text{m}
\]

\( R_{dt\_SW1} = 62.4314 \text{ km} \)

\[
R_{dt\_SW3} := \sqrt[4]{\frac{D_0}{D_3}} \cdot R_{dt\_SW0} = 7.9014 \times 10^4 \text{m}
\]

\( R_{dt\_SW3} = 79.0135 \text{ km} \)

As expected; since the detection thresholds for the Swerling 1, 2, 3, and 4 targets is higher than for a Swerling 0 target, the detection ranges for the other Swerling cases are less than for a Swerling 0 target.
Exercise 3-5  A radar system has a single pulse signal-to-noise ratio \( SNR = -2 \) dB. If the required probability of false alarm \( P_{fa} = 10^{-6} \), what is the probability of detection, \( P_d \) (no units), associated with the single pulse SNR? If the radar system has an integration time \( T_I = 40 \) msec and pulse repetition frequency \( PRF = 800 \) Hz, how many pulses are integrated, \( n_p \)? What is the coherent integration gain, \( G_I \) (no units and dB), the signal-to-noise ratio after coherent integration, \( SNR_n \) (no units and dB), and the probability of detection, \( P_d \) (no units), associated with the \( SNR_n \) after coherent integration? What is the noncoherent integration gain, \( G_I \) (no units and dB), using Marcum's equation, the signal-to-noise ratio after noncoherent integration, \( SNR_n \) (no units and dB), and the probability of detection, \( P_d \) (no units), associated with the \( SNR_n \) after noncoherent integration? Use Table 3-1 for the relationship between the S/N required for detection, \( P_d \), and \( P_{fa} \).

We start by defining the values for all the terms.

\[
SNR\_dB \equiv -2 \quad T_I \equiv 40 \times 10^{-3} \cdot \text{sec} \quad PRF \equiv 800 \cdot \text{Hz}
\]

We start by using Table 3-1 to determine the \( P_d \) for the given \( P_{fa} \) and \( SNR\_dB \) (dB). We then use Equation (3-35) to compute the coherent integration gain, \( G_I \) (no units). Next we convert the coherent integration gain to decibels, \( G_{I\_dB} \) (dB). The S/N after the coherent integration is the sum (since both values are in decibels) of the single pulse S/N and the coherent integration gain, \( SNR_n\_dB \) (dB), as given in Equation (3-34). We finish up by looking up the \( P_d \) that corresponds to the S/N after coherent integration and \( P_{fa} = 10^{-6} \) using Table 3-1.

\[P_d < 0.05 \quad \text{Table 3-1 for SNR = -2 dB and } P_{fa} = 10^{-6}\]

\[n_p := T_I \cdot PRF = 32\]

\[G_I := n_p = 32\]

\[G_{I\_dB} := 10 \log(G_I) = 15.0515\]

\[SNR := 10 \left( \frac{SNR\_dB}{10} \right) = 0.631\]
\[ \text{SNR}_n := G_I \cdot \text{SNR} = 20.1906 \]
\[ = 32 \cdot 0.6309573448019325 = 20.1906 \]

\[ \text{SNR}_{n \_dB} := G_{I \_dB} + \text{SNR}_{\_dB} = 13.0515 \]
\[ = 15.051499783199061 + -2 = 13.0515 \]

\[ 0.85 < P_d < 0.90 \quad \text{Table 3-1 for SNR} = 13.05 \text{ dB and } P_{fa} = 10^{-6} \]

We use Equation (3-37), Marcum, to compute the noncoherent integration gain, \( G_I \) (no units). Next we convert the noncoherent integration gain to decibels, \( G_{I \_dB} \) (dB). The S/N after the noncoherent integration is the sum (since both values are in decibels) of the single pulse S/N and the noncoherent integration gain, \( \text{SNR}_{n \_dB} \) (dB), as given in Equation (3-34). We finish up by looking up the \( P_d \) that corresponds to the S/N after noncoherent integration and \( P_{fa} = 10^{-6} \) using Table 3-1.

\[ G_I := n_p^{0.76} = 13.9288 \]
\[ = 32^{0.76} = 13.9288 \]

\[ G_{I \_dB} := 10 \log(G_I) = 11.4391 \]
\[ = 10 \log (13.928809012737986) = 11.4391 \]

\[ \text{SNR}_n := G_I \cdot \text{SNR} = 8.7885 \]
\[ = 13.928809012737986 \cdot 0.6309573448019325 = 8.7885 \]

\[ \text{SNR}_{n \_dB} := G_{I \_dB} + \text{SNR}_{\_dB} = 9.4391 \]
\[ = 11.439139835231284 + -2 = 9.4391 \]

\[ 0.15 < P_d < 0.20 \quad \text{Table 3-1 for SNR} = 9.44 \text{ dB and } P_{fa} = 10^{-6} \]
Exercise 3-6  In order to increase the performance of a power-limited radar system, you plan to use multiple pulse integration. You currently achieve a single-pulse $P_d = 0.5$ and $P_{fa} = 10^{-4}$. How many coherent pulses are required to be integrated to obtain $P_d = 0.95$ and $P_{fa} = 10^{-12}$? How many noncoherent pulses are required to be integrated to obtain the new $P_d$ and $P_{fa}$? Use Marcum's optimum noncoherent integration gain.

Currently the $S/N$ required is given by $SNR_{dB}$ (dB). To achieve the new $P_d$ and $P_{fa}$ values the $S/N$ required is given by $SNR_{n\_dB}$ (dB). The difference between these two $S/N$ values is the necessary coherent integration gain, $GI_{dB}$ (dB), as shown in Equation 3-34 converted to decibels.

$$SNR_{n} = GI \cdot SNR$$

$$SNR_{n\_dB} = GI_{dB} + SNR_{dB}$$

Using Figure 3-9 or Table 3-1 we can find the $S/N$ required to achieve the given $P_d$ and $P_{fa}$ values.

$$SNR_{dB} = 9.4$$

$$SNR_{n\_dB} = 16.1$$

$$GI_{dB} = SNR_{n\_dB} - SNR_{dB} = 6.7$$

The relationship between coherent integration gain, $GI$ (no units), the number of coherent pulses integrated, $n_p$ (pulses), is given Equation 3-35. Thus, the number of coherent pulses required is determined by converting the coherent integration gain from dB to absolute, as shown below:

$$GI = n_p$$

$$n_p = 10 \left( \frac{GI_{dB}}{10} \right) = 4.6774 \text{ pulses}$$

Since pulses can only have integer values, we would say that 5 coherent pulses are required.

Marcum's optimum noncoherent integration gain, $GI$ (no units), as a function of the number of pulses integrated is given in Equation 3-37. The number of noncoherent pulses is determined as shown below:
\[ G_I = n_p^{0.76} \]

\[ \ln(G_I) = 0.76 \cdot \ln(n_p) \]

\[ \ln(n_p) = \frac{\ln(G_I)}{0.76} \]

\[ G_I := 10^{\left(\frac{G_{I \, \text{dB}}}{10}\right)} = 4.6774 \]

\[ n_p := e^{\left(\frac{\ln(G_I)}{0.76}\right)} = 7.6134 \]

Since pulses can only have integer values, we would say that 8 noncoherent pulses are required.
**Exercise 3-7** A radar system requires the following detection criteria $P_d = 0.5$ and $P_{fa} = 10^{-6}$. The radar system has the following characteristics: antenna half-power (-3 dB) beamwidth $\theta_{3dB} = 0.96$ degrees, antenna scan rate $\theta_{dot} = 36$ degrees/second, and pulse repetition frequency PRF = 1200 Hz. What is the noncoherent integration gain for Swerling Case 0, 1, 2, 3, and 4 targets? What is the detection range factor (detection range relative to that for a Swerling Case 0 target) for each Swerling Case? Assume the noncoherent integration gain and detection threshold are matched to each Swerling Case. Compute the results using the equations in Section 3.3.1 and then compare them to the results obtained in Figure 3-14 and Table 3-5. [Hint: use the results of Exercise 3-3]

We start by defining the values for all the terms.

$$P_d \equiv 0.5 \quad P_{fa} \equiv 10^{-6} \quad \theta_{3dB} \equiv 0.96 \text{ deg} \quad \theta_{dot} \equiv 36 \frac{\text{deg}}{\text{sec}} \quad \text{PRF} \equiv 1200 \text{ Hz}$$

**Note:** this solution uses North's equations using the Q probability integral (qnorm in Mathcad). The solution in Exercise 3-7-1 uses Albersheim's equation. As we will see the results from both equations are almost identical.

We start by using the results of Exercise 3-3: additional signal-to-noise ratio relative to a Swerling 0 target required for detection of a Swerling 1 target, $\Delta \text{SNR}_{f1}$ (no units); detection threshold for a Swerling 0 target, $D_0$ (no units); detection threshold for a Swerling 1 target, $D_1$ (no units); detection threshold for a Swerling 2 target, $D_2$ (no units); detection threshold for a Swerling 3 target, $D_3$ (no units); and detection threshold for a Swerling 4 target, $D_4$ (no units).

$$\Delta \text{SNR}_{f1} \equiv 1.4218 \quad D_3 \equiv 15.8771 \quad D_4 := D_3 = 15.8771 \quad D_0 \equiv 13.3155 \quad D_1 \equiv 18.9316 \quad D_2 := D_1 = 18.9316$$

The noncoherent integration gain is a function of the number of pulses integrated, $n_p$ (no units). The number of pulses integrated is a function of the integration time and the pulse repetition frequency, as given in Equation (3-41). For a scanning radar system, the integration time is the target illumination time, $T_{ill}$ (seconds), as given by Equation (3-42).

$$T_{ill} := \frac{\theta_{3dB}}{\theta_{dot}} = 0.0267 \text{ s}$$
\[ n_p := \frac{T_{\text{PRF}}}{T_{\text{PRF}}} = 32 \]

= 0.0267 \cdot \text{sec} \cdot (1200 \cdot \text{Hz}) = 32.04

For a Swerling 0 target, the noncoherent integration gain, \( G_{I0} \) (no units) and \( G_{I0 \_dB} \) (dB), as given in Equation (3-49) is a function of the equivalent single-pulse signal-to-noise ratio required for detection after noncoherent integration of multiple pulses, \( D_{0n} \) (no units), as given in Equation (3-46), and the ideal signal-to-noise ratio required for coherent detection, \( D_c \) (no units), as given in Equation (3-47).

\[
D_c := \frac{1}{2} \left( \text{qnorm}(1 - P_{fa}, 0, 1) - \text{qnorm}(1 - P_d, 0, 1) \right)^2 = 11.2975
\]

\[
D_{0n} := \frac{D_c}{2 \cdot n_p} \left( 1 + \sqrt{1 + \frac{9.2 \cdot n_p}{D_c}} \right) = 1.0948
\]

\[
= \frac{11.297521329832369 \cdot \left( 1 + \sqrt{1 + \frac{9.2 \cdot 31.999999999999}{11.297521329832369}} \right)}{(2 \cdot 31.999999999999996)} = 1.0948
\]

\[
G_{I0} := \frac{D_0}{D_{0n}} = 12.1629
\]

\[
= \frac{13.3155}{1.0947666510840586} = 12.1629
\]

\[
10 \log(G_{I0}) = 10.8504
\]

\[
= 10 \cdot \log (12.162865928382766) = 10.8504
\]

The noncoherent integration gain for Swerling 1, 2, 3, and 4 targets: \( G_{I1}, G_{I2}, G_{I3}, \) and \( G_{I4}, \) respectively, are given in Equations (3-50) and (3-51).

\[
G_{I1} := G_{I0} \cdot \left( \Delta \text{SNR}_{f1} \right)^{(-0.03 \log(n_p))} = 11.9711
\]

\[
= 12.162865928382766 \cdot 1.4218 \left[ (-0.03) \cdot \log (31.999999999999) \right] = 11.9711
\]

\[
10 \log(G_{I1}) = 10.7813
\]

\[
= 10 \cdot \log (11.971114180749947) = 10.7813
\]
\[ G_{I2} := G_{I0} \cdot (\Delta \text{SNR}_{f1}) \left( 1 - \frac{1+0.03 \log(n_p)}{n_p} \right) = 17.0955 \]

\[
= 12.162865928382766 \cdot 1.4218 \left[ 1 - \frac{(1+0.03 \cdot \log(31.999999999999996))}{31.999999999999996} \right] = 17.0955
\]

\[
10 \log(G_{I2}) = 12.3288 = 10 \cdot \log(17.095529585154594) = 12.3288
\]

\[ G_{I3} := G_{I0} \cdot (\Delta \text{SNR}_{f1}) \left( -\frac{0.03 \log(n_p)}{2} \right) = 12.0666 \]

\[
= 12.162865928382766 \cdot 1.4218 \left[ \frac{-0.03 \cdot \log(31.999999999999996)}{2} \right] = 12.0666
\]

\[
10 \log(G_{I3}) = 10.8159 = 10 \cdot \log(12.066609167194539) = 10.8159
\]

\[ G_{I4} := G_{I0} \cdot (\Delta \text{SNR}_{f1}) \left( \frac{1}{2} \frac{1+0.03 \log(n_p)}{2 \cdot n_p} \right) = 14.4198 \]

\[
= 12.162865928382766 \cdot 1.4218 \left[ \frac{1}{2} \frac{(1+0.03 \cdot \log(31.999999999999996))}{(2 \cdot 31.999999999999996)} \right] = 14.4198
\]

\[
10 \log(G_{I4}) = 11.5896 = 10 \cdot \log(14.419800079021082) = 11.5896
\]

We can read the same values from Figure 3-14 and Table 3-5.

Equation 3-43 defines the radar detection range. We write this equation for the two detection ranges, integration gains, and detection thresholds. We can see from these two equations that the vast majority of terms are the same. These same terms can be represented by the constant C. We can then solve for C in each equation, and then equate the two. We can then solve for the new detection range as a function of the integration gains.
and detection thresholds. These equations are shown below.

\[
R_{dt1} = \sqrt{\frac{P_R \cdot G_{RT} \cdot \lambda \cdot \sigma \cdot G_{sp} \cdot G_{I1}}{(4\pi)^3 \cdot SNR_{dt1} \cdot P_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}}
\]

\[
R_{dt2} = \sqrt{\frac{P_R \cdot G_{RT} \cdot \lambda \cdot \sigma \cdot G_{sp} \cdot G_{I2}}{(4\pi)^3 \cdot SNR_{dt2} \cdot P_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}}
\]

\[
C = R_{dt1} \cdot \frac{SNR_{dt1}}{G_{I1}} = R_{dt2} \cdot \frac{SNR_{dt2}}{G_{I2}}
\]

\[
R_{dt2} = \frac{C \cdot G_{I2}}{\sqrt{SNR_{dt2}}}
\]

\[
R_{dt1} = \frac{C \cdot G_{I1}}{\sqrt{SNR_{dt1}}}
\]

We can use this equation to compute the detection range factor for Swerling 1, 2, 3, and 4 targets: \(\text{Range\_Factor}_{SW1}\), \(\text{Range\_Factor}_{SW2}\), \(\text{Range\_Factor}_{SW3}\), and \(\text{Range\_Factor}_{SW4}\), respectively.

\[
\text{Range\_Factor}_{SW1} := \frac{G_{I1} \cdot D_0}{\sqrt{G_{I0} \cdot D_1}} \cdot (1) = 0.9122
\]

\[
\text{Range\_Factor}_{SW2} := \frac{G_{I2} \cdot D_0}{\sqrt{G_{I0} \cdot D_2}} \cdot (1) = 0.9971
\]

\[
\text{Range\_Factor}_{SW3} := \frac{G_{I3} \cdot D_0}{\sqrt{G_{I0} \cdot D_3}} \cdot (1) = 0.9551
\]

\[
\text{Range\_Factor}_{SW4} := \frac{G_{I4} \cdot D_0}{\sqrt{G_{I0} \cdot D_4}} \cdot (1) = 0.9986
\]

We can read the same values from Table 3-5.
Exercise 3-7-1 A radar system requires the following detection criteria $P_d = 0.5$ and $P_{fa} = 10^{-6}$. The radar system has the following characteristics: antenna half-power (-3 dB) beamwidth $\theta_{3dB} = 0.96$ degrees, antenna scan rate $\theta_{dot} = 36$ degrees/second, and pulse repetition frequency PRF = 1200 Hz. What is the noncoherent integration gain for Swerling Case 0, 1, 2, 3, and 4 targets? What is the detection range factor (detection range relative to that for a Swerling Case 0 target) for each Swerling Case? Assume the noncoherent integration gain and detection threshold are matched to each Swerling Case. Compute the results using the equations in Section 3.3.1 and then compare them to the results obtained in Figure 3-14 and Table 3-5. [Hint: use the results of Exercise 3-3-1]

We start by defining the values for all the terms.

$P_d \equiv 0.5 \quad P_{fa} \equiv 10^{-6} \quad \theta_{3dB} \equiv 0.96 \cdot \text{deg} \quad \theta_{dot} \equiv 36 \cdot \frac{\text{deg}}{\text{sec}} \quad \text{PRF} \equiv 1200 \cdot \text{Hz}$

Note: this solution uses Albersheim's equation. The solution in Exercise 3-7 uses North's equations using the Q probability integral. As we will see the results from both equations are almost identical.

We start by using the results of Exercise 3-3-1: additional signal-to-noise ratio relative to a Swerling 0 target required for detection of a Swerling 1 target, $\Delta \text{SNR}_{f1}$ (no units); detection threshold for a Swerling 0 target, $D_0$ (no units); detection threshold for a Swerling 1 target, $D_1$ (no units); detection threshold for a Swerling 2 target, $D_2$ (no units); detection threshold for a Swerling 3 target, $D_3$ (no units); and detection threshold for a Swerling 4 target, $D_4$ (no units).

$D_0 \equiv 13.3375 \quad D_1 \equiv 18.9316 \quad D_2 := D_1 = 18.9316 \quad \Delta \text{SNR}_{f1} \equiv 1.4194 \quad D_3 \equiv 15.8902 \quad D_4 := D_3 = 15.8902$

The noncoherent integration gain is a function of the number of pulses integrated, $n_p$ (no units). The number of pulses integrated is a function of the integration time and the pulse repetition frequency, as given in Equation (3-41). For a scanning radar system, the integration time is the target illumination time, $T_{ill}$ (seconds), as given by Equation (3-42).

$T_{ill} := \frac{\theta_{3dB}}{\theta_{dot}} = 0.0267 \text{s}$
For a Swerling 0 target, the noncoherent integration gain, $G_{I0}$ (no units) and $G_{I0\_dB}$ (dB), as given in Equation (3-49) is a function of the equivalent single-pulse signal-to-noise ratio required for detection after noncoherent integration of multiple pulses, $D_{0n}$ (no units) and $D_{0n\_dB}$ (dB), as given in Equations (3-48) and (3-26).

\[
\begin{align*}
A &:= \ln \left( \frac{0.62}{P_{fa}} \right) = 13.3375 \\
B &:= \ln \left( \frac{P_d}{1 - P_d} \right) = 0 \\
D_{0n\_dB} &:= -5 \log(n_p) + \left( 6.2 + \frac{4.54}{\sqrt{n_p + 0.44}} \right) \log (A + 0.12 \cdot A \cdot B + 1.7 \cdot B) = 0.3465 \\
\end{align*}
\]

\[
\begin{align*}
D_{0n} &:= 10^{\frac{D_{0n\_dB}}{10}} = 1.0831 \\
G_{I0} &:= \frac{D_0}{D_{0n}} = 12.3147 \\
10 \log(G_{I0}) &:= 10.9042 \\
\end{align*}
\]

The noncoherent integration gain for Swerling 1, 2, 3, and 4 targets: $G_{I1}$, $G_{I2}$, $G_{I3}$, and $G_{I4}$, respectively, are given in Equations (3-50) and (3-51).

\[
G_{I1} := G_{I0} \cdot \left( \Delta \text{SNR}_{f1} \right)^{-0.03 \log(n_p)} = 12.1215
\]
\[
10 \log (G_{I1}) = 10.8356
\]
\[
= 10 \cdot \log (12.121473273732049) = 10.8356
\]
\[
\begin{align*}
G_{I2} & := G_{I0} \cdot (\Delta SNR_f) \left(1 + 0.03 \log(n_p)\right) / n_p \bigg) = 17.2807 \\
& = 12.314693988404999 \cdot 1.4194 \left[1 - \frac{(1+0.03 \log(31.999999999999996))}{31.999999999999996}\right] = 17.2807
\end{align*}
\]
\[
10 \log (G_{I2}) = 12.3756
\]
\[
= 10 \cdot \log (17.280667681144692) = 12.3756
\]
\[
\begin{align*}
G_{I3} & := G_{I0} \cdot (\Delta SNR_f) \left(-0.03 \log(n_p)\right) / 2) = 12.2177 \\
& = 12.314693988404999 \cdot 1.4194 \left[(-0.03 \log(31.999999999999996)) / 2 \right] = 12.2177
\end{align*}
\]
\[
10 \log (G_{I3}) = 10.8699
\]
\[
= 10 \cdot \log (12.217701668261503) = 10.8699
\]
\[
\begin{align*}
G_{I4} & := G_{I0} \cdot (\Delta SNR_f) \left(\frac{1}{2} + \frac{1+0.03 \log(n_p)}{2 \cdot n_p}\right) = 14.5879 \\
& = 12.314693988404999 \cdot 1.4194 \left[\frac{1}{2} \left(1+0.03 \log(31.999999999999996)\right) / (2 \cdot 31.999999999999996)\right] = 14.5879
\end{align*}
\]
\[
10 \log (G_{I4}) = 11.6399
\]
\[
= 10 \cdot \log (14.587876281646246) = 11.6399
\]
We can read the same values from Figure 3-14 and Table 3-5.

Equation 3-43 defines the radar detection range. We write this equation for the two detection ranges, integration gains, and detection thresholds. We can see from these two equations that the vast majority of terms are the same. These same terms can be represented by the constant C. We can then solve for C in each equation, and then equate the two. We can then solve for the new detection range as a function of the integration gains and detection thresholds. These equations are shown below.

\[ R_{dt1} = \sqrt{\frac{F_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot GI_1}{(4\pi)^3 \cdot SNR_{dt1} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}} \]

\[ R_{dt2} = \sqrt{\frac{F_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot GI_2}{(4\pi)^3 \cdot SNR_{dt2} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}} \]

\[ C = R_{dt1} \cdot \frac{SNR_{dt1}}{GI_1} = R_{dt2} \cdot \frac{SNR_{dt2}}{GI_2} \]

We can use this equation to compute the detection range factor for Swerling 1, 2, 3, and 4 targets: Range\_Factor\_SW1, Range\_Factor\_SW2, Range\_Factor\_SW3, and Range\_Factor\_SW4, respectively.

\[ \text{Range\_Factor\_SW1} := \frac{G_{I1} \cdot D_0}{\sqrt{G_{I0} \cdot D_1}} \cdot (1) = 0.9125 \]

\[ \text{Range\_Factor\_SW2} := \frac{G_{I2} \cdot D_0}{\sqrt{G_{I0} \cdot D_2}} \cdot (1) = 0.9971 \]

\[ \text{Range\_Factor\_SW3} := \frac{G_{I3} \cdot D_0}{\sqrt{G_{I0} \cdot D_3}} \cdot (1) = 0.9553 \]

\[ \text{Range\_Factor\_SW4} := \frac{G_{I4} \cdot SNR_{dt1}}{G_{II} \cdot SNR_{dt2}} \cdot R_{dt1} \]

\[ = \frac{G_{I2} \cdot SNR_{dt1}}{G_{I1} \cdot SNR_{dt2}} \cdot R_{dt1} \]
Range\_Factor_{SW4} := \frac{G_{14} \cdot D_0}{G_{10} \cdot D_4} \cdot (1) = 0.9986
\begin{align*}
\sqrt{\frac{14.5878762816246 \cdot 13.3375}{(12.314693988404999 \cdot 15.8902)}} \cdot 1 &= 0.9986
\end{align*}

We can read the same values from Table 3-5.
Exercise 3-8 A radar system requires the following detection criteria $P_d = 0.9$ and $P_{fa} = 10^{-8}$. The radar system has the following characteristics:

antenna half-power (-3 dB) beamwidth $\theta_{3dB} = 0.96$ degrees, antenna scan rate $\theta_{dot} = 36$ degrees/second, and pulse repetition frequency $PRF = 1200$ Hz. What is the noncoherent integration gain for Swerling Case 0, 1, 2, 3, and 4 targets? What is the detection range factor (detection range relative to that for a Swerling Case 0 target) for each Swerling Case? Assume the noncoherent integration gain and detection threshold are matched to each Swerling Case. Compute the results using the equations in Section 3.3.1 and then compare them to the results obtained in Figure 3-15 and Table 3-5. [Hint: use the results of Exercise 3-4]

We start by defining the values for all the terms.

\[
P_d \equiv 0.9 \quad P_{fa} \equiv 10^{-8} \quad \theta_{3dB} \equiv 0.96 \cdot \text{deg} \quad \theta_{dot} \equiv 36 \frac{\text{deg}}{\text{sec}} \quad PRF \equiv 1200 \cdot \text{Hz}
\]

Note: this solution uses North's equations using the Q probability integral (qnorm in Mathcad). The solution in Exercise 3-8-1 uses Albersheim's equation. As we will see the results from both equations are almost identical.

We start by using the results of Exercise 3-4: additional signal-to-noise ratio relative to a Swerling 0 target required for detection of a Swerling 1 target, $\Delta SNR_{f1}$ (no units); detection threshold for a Swerling 0 target, $D_0$ (no units); detection threshold for a Swerling 1 target, $D_1$ (no units); detection threshold for a Swerling 2 target, $D_2$ (no units); detection threshold for a Swerling 3 target, $D_3$ (no units); and detection threshold for a Swerling 4 target, $D_4$ (no units).

\[
D_0 \equiv 26.5205 \quad D_1 \equiv 173.8348 \quad D_2 := D_1 = 173.8348 \quad \Delta SNR_{f1} \equiv 6.5547 \quad D_3 \equiv 67.8984 \quad D_4 := D_3 = 67.8984
\]

The noncoherent integration gain is a function of the number of pulses integrated, $n_p$ (no units). The number of pulses integrated is a function of the integration time and the pulse repetition frequency, as given in Equation (3-41). For a scanning radar system, the integration time is the target illumination time, $T_{ill}$ (seconds), as given by Equation (3-42).

\[
T_{ill} := \frac{\theta_{3dB}}{\theta_{dot}} = 0.0267 \text{ s}
\]

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For a Swerling 0 target, the noncoherent integration gain, $G_{I0}$ (no units) and $G_{I0\ dB}$ (dB), as given in Equation (3-49) is a function of the equivalent single-pulse signal-to-noise ratio required for detection after noncoherent integration of multiple pulses, $D_{0n}$ (no units), as given in Equation (3-46), and the ideal signal-to-noise ratio required for coherent detection, $D_c$ (no units), as given in Equation (3-47).

\[
D_c := \frac{1}{2} \left( \text{qnorm}(1-P_{fa}, 0, 1) - \text{qnorm}(1-P_d, 0, 1) \right)^2 = 23.7605
\]

\[
D_{0n} := \frac{D_c}{2 \cdot n_p} \left( 1 + \sqrt{1 + \frac{9.2 \cdot n_p}{D_c}} \right) = 1.7298
\]

\[
G_{I0} := \frac{D_0}{D_{0n}} = 15.3316
\]

\[
10 \log(G_{I0}) = 11.8559 = 10 \cdot \log (15.331587359739912) = 11.8559
\]

The noncoherent integration gain for Swerling 1, 2, 3, and 4 targets: $G_{I1}$, $G_{I2}$, $G_{I3}$, and $G_{I4}$, respectively, are given in Equations (3-50) and (3-51).

\[
G_{I1} := G_{I0} \cdot \left( \Delta \text{SNR}_{f1} \right)^{(-0.03 \log(n_p))} = 14.0837
\]

\[
= 15.331587359739912 \cdot 6.5547 \left[ (-0.03) \cdot \log (31.99999999999999) \right] = 14.0837
\]

\[
10 \log(G_{I1}) = 11.4872 = 10 \cdot \log (14.083678036687605) = 11.4872
\]
\[ G_{I2} := G_{I0} \cdot \left( \Delta SNR_{f1} \right)^{\left( 1 \frac{1 + 0.03 \log(n_p)}{n_p} \right)} = 94.5084 \]

\[ = 15.331587359739912 \cdot 6.5547 \cdot \left[ 1 - \frac{(1 + 0.03 \cdot \log(31.999999999999996))}{31.999999999999996} \right]^2 = 94.5084 \]

\[ 10 \log(G_{I2}) = 19.7547 \]

\[ = 10 \cdot \log(94.5084076611965) = 19.7547 \]

\[ G_{I3} := G_{I0} \cdot \left( \Delta SNR_{f1} \right)^{\left( -0.03 \log(n_p) \right)} = 14.6944 \]

\[ = 15.331587359739912 \cdot 6.5547 \cdot \left[ (-0.03) \cdot \log(31.999999999999996) \right]^2 = 14.6944 \]

\[ 10 \log(G_{I3}) = 11.6715 \]

\[ = 10 \cdot \log(14.694391452725299) = 11.6715 \]

\[ G_{I4} := G_{I0} \cdot \left( \Delta SNR_{f1} \right)^{\left( \frac{1 + 0.03 \log(n_p)}{2 \cdot n_p} \right)} = 38.0653 \]

\[ = 15.331587359739912 \cdot 6.5547 \cdot \left[ \frac{1}{2} \cdot \frac{(1 + 0.03 \cdot \log(31.999999999999996))}{(2 \cdot 31.999999999999996)} \right]^2 = 38.0653 \]

\[ 10 \log(G_{I4}) = 15.8053 \]

\[ = 10 \cdot \log(38.065258547493762) = 15.8053 \]

We can read the same values from Figure 3-15 and Table 3-5.

Equation 3-43 defines the radar detection range. We write this equation for the two detection ranges, integration gains, and detection thresholds. We can see from these two equations that the vast majority of terms are the same. These same terms can be represented by the constant \( C \). We can then solve for \( C \) in each equation, and then equate the two. We can then solve for the new detection range as a function of the integration gains and detection thresholds. These equations are shown below.
We can use this equation to compute the detection range factor for Swerling 1, 2, 3, and 4 targets: Range_FactorSW1, Range_FactorSW2, Range_FactorSW3, and Range_FactorSW4, respectively.

\[
\text{Range}_1 = 4 \sqrt{\frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_{I1}}{(4\pi)^3 \cdot \text{SNR}_{dt1} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}}
\]

\[
\text{Range}_2 = 4 \sqrt{\frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_{I2}}{(4\pi)^3 \cdot \text{SNR}_{dt2} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}}
\]

\[
C = \text{Range}_1 \cdot \frac{\text{SNR}_{dt1}}{G_{I1}} = \text{Range}_2 \cdot \frac{\text{SNR}_{dt2}}{G_{I2}}
\]

We can read the same values from Table 3-5.
**Exercise 3-8-1** A radar system requires the following detection criteria \( P_d = 0.9 \) and \( P_{fa} = 10^{-8} \). The radar system has the following characteristics: antenna half-power (-3 dB) beamwidth \( \theta_{3dB} = 0.96 \) degrees, antenna scan rate \( \theta_{dot} = 36 \) degrees/second, and pulse repetition frequency PRF = 1200 Hz. What is the noncoherent integration gain for Swerling Case 0, 1, 2, 3, and 4 targets? What is the detection range factor (detection range relative to that for a Swerling Case 0 target) for each Swerling Case? Assume the noncoherent integration gain and detection threshold are matched to each Swerling Case. Compute the results using the equations in Section 3.3.1 and then compare them to the results obtained in Figure 3-15 and Table 3-5. [Hint: use the results of Exercise 3-4-1]

We start by defining the values for all the terms.

\[
P_d \equiv 0.9 \quad P_{fa} \equiv 10^{-8} \quad \theta_{3dB} \equiv 0.96 \cdot \text{deg} \quad \theta_{dot} \equiv 36 \cdot \frac{\text{deg}}{\text{sec}} \quad \text{PRF} \equiv 1200 \cdot \text{Hz}
\]

Note: this solution uses Albersheim's equation. The solution in Exercise 3-8 uses North's equations using the Q probability integral. As we will see the results from both equations are almost identical.

We start by using the results of Exercise 3-4-1: additional signal-to-noise ratio relative to a Swerling 0 target required for detection of a Swerling 1 target, \( \Delta \text{SNR}_{f1} \) (no units); detection threshold for a Swerling 0 target, \( D_0 \) (no units); detection threshold for a Swerling 1 target, \( D_1 \) (no units); detection threshold for a Swerling 2 target, \( D_2 \) (no units); detection threshold for a Swerling 3 target, \( D_3 \) (no units); and detection threshold for a Swerling 4 target, \( D_4 \) (no units).

\[
D_0 \equiv 26.4088 \quad D_1 \equiv 173.8348 \quad D_2 := D_1 = 173.8348 \quad \Delta \text{SNR}_{f1} \equiv 6.5825 \quad D_3 \equiv 67.7552 \quad D_4 := D_3 = 67.7552
\]

The noncoherent integration gain is a function of the number of pulses integrated, \( n_p \) (no units). The number of pulses integrated is a function of the integration time and the pulse repetition frequency, as given in Equation (3-41). For a scanning radar system, the integration time is the target illumination time, \( T_{ill} \) (seconds), as given by Equation (3-42).

\[
T_{ill} := \frac{\theta_{3dB}}{\theta_{dot}} = 0.0267 \text{ s}
\]

\[
= 0.96 \cdot \text{deg} = 0.0267 \text{ s}
\]

\[
36 \cdot \left( \frac{\text{deg}}{\text{sec}} \right)
\]

\[
= 0.0267 \text{ s}
\]
For a Swerling 0 target, the noncoherent integration gain, $G_{10}$ (no units) and $G_{10_{\text{dB}}}$ (dB), as given in Equation (3-49) is a function of the equivalent single-pulse signal-to-noise ratio required for detection after noncoherent integration of multiple pulses, $D_{0n}$ (no units) and $D_{0n_{\text{dB}}}$ (dB), as given in Equations (3-48) and (3-26).

\[
A := \ln \left( \frac{0.62}{P_{fa}} \right) = 17.9426
\]
\[
= \ln \left( \frac{0.62}{10^{-8}} \right) = 17.9426
\]

\[
B := \ln \left( \frac{P_d}{1 - P_d} \right) = 2.1972
\]
\[
= \ln \left[ \frac{0.9}{(1 - 0.9)} \right] = 2.1972
\]

\[
D_{0n_{\text{dB}}} := -5 \log(n_p) + \left( 6.2 + \frac{4.54}{\sqrt{n_p + 0.44}} \right) \cdot \log(A + 0.12 \cdot A \cdot B + 1.7 \cdot B) = 2.4224
\]

\[
-5 \log(32) + \left( 6.2 + \frac{4.54}{\sqrt{32 + 0.44}} \right) \cdot \log [17.9426 + (0.12) \cdot (17.9426) \cdot (2.1972) + (1.7) \cdot (2.1972)] = 2.4224
\]

\[
D_{0n} := 10 \left( \frac{D_{0n_{\text{dB}}}}{10} \right) = 1.7468
\]
\[
= 10 \left( \frac{2.4223756057695702}{10} \right) = 1.7468
\]

\[
G_{10} := \frac{D_0}{D_{0n}} = 15.1186
\]
\[
= \frac{26.4088}{1.746777384883107} = 15.1186
\]

\[
10 \log(G_{10}) = 11.7951
\]
\[
= 10 \cdot \log (15.118583643540392) = 11.7951
\]

The noncoherent integration gain for Swerling 1, 2, 3, and 4 targets: $G_{11}$, $G_{12}$, $G_{13}$, and $G_{14}$, respectively, are given in Equations (3-50) and (3-51).
\[ G_{I1} := G_{I0} \cdot (\Delta \text{SNR}_{f1})^{(-0.03 \log(n_p))} = 13.8854 \]

\[ = 15.118583643540392 \cdot \Delta \text{SNR}_{f1}^{(-0.03) \cdot \log(n_p)} = 13.8854 \]

\[ 10 \log(G_{I1}) = 11.4256 \]

\[ = 10 \cdot \log(13.885357860584128) = 11.4256 \]

\[ G_{I2} := G_{I0} \cdot (\Delta \text{SNR}_{f1})^{\left(\frac{1+0.03 \log(n_p)}{n_p}\right)} = 93.5777 \]

\[ = 15.118583643540392 \cdot 6.5825 \cdot \frac{1+0.03 \cdot \log(31.999999999999996)}{31.999999999999996} = 93.5777 \]

\[ 10 \log(G_{I2}) = 19.7117 \]

\[ = 10 \cdot \log(93.5777172474386) = 19.7117 \]

\[ G_{I3} := G_{I0} \cdot (\Delta \text{SNR}_{f1})^{\frac{-0.03 \log(n_p)}{2}} = 14.4889 \]

\[ = 15.118583643540392 \cdot 6.5825 \cdot \frac{-0.03 \cdot \log(31.999999999999996)}{2} = 14.4889 \]

\[ 10 \log(G_{I3}) = 11.6103 \]

\[ = 10 \cdot \log(14.48855863584682) = 11.6103 \]

\[ G_{I4} := G_{I0} \cdot (\Delta \text{SNR}_{f1})^{\left(\frac{1}{2} \cdot \frac{1+0.03 \log(n_p)}{2 \cdot n_p}\right)} = 37.6133 \]

\[ = 15.118583643540392 \cdot 6.5825 \cdot \frac{1}{2} \cdot \frac{1+0.03 \cdot \log(31.999999999999996)}{2 \cdot 31.999999999999996} = 37.6133 \]

\[ 10 \log(G_{I4}) = 15.7534 \]

\[ = 10 \cdot \log(37.613329357781836) = 15.7534 \]
We can read the same values from Figure 3-15 and Table 3-5.

Equation 3-43 defines the radar detection range. We write this equation for the two detection ranges, integration gains, and detection thresholds. We can see from these two equations that the vast majority of terms are the same. These same terms can be represented by the constant C. We can then solve for C in each equation, and then equate the two. We can then solve for the new detection range as a function of the integration gains and detection thresholds. These equations are shown below.

\[
R_{dt1} = \sqrt{\frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_{I1}}{(4\pi)^3 \cdot SNR_{dt1} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}}
\]

\[
R_{dt2} = \sqrt{\frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_{I2}}{(4\pi)^3 \cdot SNR_{dt2} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}}
\]

\[
C = \frac{R_{dt1} \cdot SNR_{dt1}}{G_{I1}} = \frac{R_{dt2} \cdot SNR_{dt2}}{G_{I2}}
\]

We can use this equation to compute the detection range factor for Swerling 1, 2, 3, and 4 targets: \(\text{Range\_Factor}_{SW1}\), \(\text{Range\_Factor}_{SW2}\), \(\text{Range\_Factor}_{SW3}\), and \(\text{Range\_Factor}_{SW4}\) respectively.

\[
\text{Range\_Factor}_{SW1} := \frac{G_{I1} \cdot D_0}{G_{I0} \cdot D_1} \cdot (1) = 0.6112
\]

\[
= \frac{4}{\sqrt{(15.118583643540392 \cdot 173.8348) \cdot 1}} = 0.6112
\]

\[
\text{Range\_Factor}_{SW2} := \frac{G_{I2} \cdot D_0}{G_{I0} \cdot D_2} \cdot (1) = 0.9847
\]

\[
= \frac{4}{\sqrt{(15.118583643540392 \cdot 173.8348) \cdot 1}} = 0.9847
\]
Range_Factor_{SW3} := \sqrt{\frac{G_{13} \cdot D_0}{G_{10} \cdot D_3}} \cdot (1) = 0.7818

Range_Factor_{SW4} := \sqrt{\frac{G_{14} \cdot D_0}{G_{10} \cdot D_4}} \cdot (1) = 0.9923

We can read the same values from Table 3-5.
**Exercise 3-9** A Swerling Case 0 target is detected by the radar system in Exercise 3-8 at a range $R_{dt} = 100$ km. If the detection threshold is unchanged, at what ranges can the radar system detect Swerling Case 1, 2, 3, and 4 targets? [Hint: use the results from Exercise 3-8]

We start by defining the values for all the terms.

$$R_{dt\_SW0} \equiv 100 \times 10^3 \cdot m$$

We will use the noncoherent integration gain values, $G_I$ (no units), for each Swerling Case: SW0, SW1, SW2, SW3, and SW4, from Exercise 3-8.

$$G_{I\_SW0} \equiv 15.3316 \quad G_{I\_SW1} \equiv 14.0837 \quad G_{I\_SW2} \equiv 94.5054 \quad G_{I\_SW3} \equiv 14.6944 \quad G_{I\_SW4} \equiv 38.0653$$

Equation 3-43 defines the radar detection range. We write this equation for the two detection ranges and integration gains. We can see from these two equations that the vast majority of terms are the same. These same terms can be represented by the constant $C$. We can then solve for $C$ in each equation, and then equate the two. We then solve for the new detection range as a function of the integration gains. These equations are shown below.

$$R_{dt1} = \sqrt[4]{\frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_I}{(4\pi)^3 \cdot \text{SNR}_{dt} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}}$$

$$R_{dt2} = \sqrt[4]{\frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_{I2}}{(4\pi)^3 \cdot \text{SNR}_{dt} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}}$$

$$C = \frac{R_{dt1}^4}{G_{I1}} = \frac{R_{dt2}^4}{G_{I2}}$$

$$R_{dt2} = \left(\frac{G_{I2}}{G_{I1}}\right)^{1/4} \cdot R_{dt1}^{1/4}$$

$$R_{dt1} = \sqrt[4]{\frac{G_{I2}}{G_{I1}}} \cdot R_{dt1}$$

We can use this equation to compute the detection range (meters) for Swerling 1, 2, 3, and 4 targets: $R_{dt\_SW1}$, $R_{dt\_SW2}$, $R_{dt\_SW3}$, and $R_{dt\_SW4}$, respectively.
\[ R_{dt,SW1} := \frac{4}{\sqrt{G_{I,SW1}}} \cdot R_{dt,SW0} = 9.79 \times 10^4 \text{ m} \]

\[ R_{dt,SW1} = 97.8999 \text{ km} \]

\[ R_{dt,SW2} := \frac{4}{\sqrt{G_{I,SW2}}} \cdot R_{dt,SW0} = 1.5757 \times 10^5 \text{ m} \]

\[ R_{dt,SW2} = 157.5677 \text{ km} \]

\[ R_{dt,SW3} := \frac{4}{\sqrt{G_{I,SW3}}} \cdot R_{dt,SW0} = 9.8944 \times 10^4 \text{ m} \]

\[ R_{dt,SW3} = 98.9444 \text{ km} \]

\[ R_{dt,SW4} := \frac{4}{\sqrt{G_{I,SW4}}} \cdot R_{dt,SW0} = 1.2553 \times 10^5 \text{ m} \]

\[ R_{dt,SW4} = 125.5265 \text{ km} \]
**Exercise 3-9-1** A Swerling Case 0 target is detected by the radar system in Exercise 3-8-1 at a range \( R_{dt} = 100 \text{ km} \). If the detection threshold is unchanged, at what ranges can the radar detect Swerling Case 1, 2, 3, and 4 targets? [Hint: use the results from Exercise 3-8-1]

We start by defining the values for all the terms.

\[
R_{dt\_SW0} \equiv 100 \times 10^3 \text{m}
\]

We will use the noncoherent integration gain values, \( G_i \) (no units), for each Swerling Case: SW0, SW1, SW2, SW3, and SW4, from Exercise 3-8-1.

\[
G_{I\_SW0} \equiv 15.1186 \quad G_{I\_SW1} \equiv 13.8854 \quad G_{I\_SW2} \equiv 93.5777 \quad G_{I\_SW3} \equiv 14.4889 \quad G_{I\_SW4} \equiv 37.6133
\]

Equation 3-43 defines the radar detection range. We write this equation for the two detection ranges and integration gains. We can see from these two equations that the vast majority of terms are the same. These same terms can be represented by the constant \( C \). We can then solve for \( C \) in each equation, and then equate the two. We then solve for the new detection range as a function of the integration gains. These equations are shown below.

\[
\begin{align*}
R_{dt1} &= \frac{4}{(4\pi)^3} \frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_{I1}}{\text{SNR}_{dt} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R} \\
R_{dt2} &= \frac{4}{(4\pi)^3} \frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_{I2}}{\text{SNR}_{dt} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R} \\
C &= \frac{R_{dt1}}{G_{I1}} = \frac{R_{dt2}}{G_{I2}} \\
R_{dt1} &= \frac{4}{G_{I1}} \sqrt{C \cdot G_{I1}} \\
R_{dt2} &= \frac{4}{G_{I2}} \sqrt{C \cdot G_{I2}} \\
R_{dt2} &= \left( \frac{G_{I2}}{G_{I1}} \right) \cdot R_{dt1} \cdot \frac{4}{G_{I2}} \\
R_{dt1} &= \sqrt{\frac{G_{I2}}{G_{I1}}} \cdot R_{dt1}
\end{align*}
\]

We can use this equation to compute the detection range (meters) for Swerling 1, 2, 3, and 4 targets: \( R_{dt\_SW1} \), \( R_{dt\_SW2} \), \( R_{dt\_SW3} \), and \( R_{dt\_SW4} \), respectively.
\[
R_{dt, SW1} := \frac{G_{I, SW1}}{G_{I, SW0}} \cdot R_{dt, SW0} = 9.7895 \times 10^4 \text{ m} = \frac{\sqrt{13.8854}}{15.1186} \cdot (100 \cdot 10^3 \text{ m}) = 9.7895 \times 10^4 \text{ m}
\]

\[
R_{dt, SW1} = 97.8953 \cdot \text{km}
\]

\[
R_{dt, SW2} := \frac{G_{I, SW2}}{G_{I, SW0}} \cdot R_{dt, SW0} = 1.5773 \times 10^5 \text{ m} = \frac{\sqrt{93.5777}}{15.1186} \cdot (100 \cdot 10^3 \text{ m}) = 1.5773 \times 10^5 \text{ m}
\]

\[
R_{dt, SW2} = 157.7303 \cdot \text{km}
\]

\[
R_{dt, SW3} := \frac{G_{I, SW3}}{G_{I, SW0}} \cdot R_{dt, SW0} = 9.8942 \times 10^4 \text{ m} = \frac{\sqrt{14.4889}}{15.1186} \cdot (100 \cdot 10^3 \text{ m}) = 9.8942 \times 10^4 \text{ m}
\]

\[
R_{dt, SW3} = 98.9421 \cdot \text{km}
\]

\[
R_{dt, SW4} := \frac{G_{I, SW4}}{G_{I, SW0}} \cdot R_{dt, SW0} = 1.2559 \times 10^5 \text{ m} = \frac{\sqrt{G_{I, SW4}}}{15.1186} \cdot (100 \cdot 10^3 \text{ m}) = 1.2559 \times 10^5 \text{ m}
\]

\[
R_{dt, SW4} = 125.5907 \cdot \text{km}
\]
**Exercise 3-10** A radar has a probability of detection \( P_d = 0.4 \), and probability of false alarm \( P_{fa} = 10^{-6} \) for each detection event. Calculate the cumulative probabilities of detection \( (P_{dc}) \) and false alarm \( (P_{fac}) \) when 10 individual detections are combined. Use Figure 3-9 or Table 3-1 to determine the detection threshold associated with each detection attempt and the detection threshold associated with the cumulative probabilities of detection and false alarm. What is the change in detection range associated with these two detection thresholds?

We start by defining the values for all the terms.

\[
N = 10 \quad P_d(x) := 0.4 \quad P_{fa}(x) := 10^{-6} \quad i := 1 \ldots N
\]

The cumulative probability of detection, \( P_{dc} \) (no units), is given in Equation 3-52. The cumulative probability of false alarm, \( P_{fac} \) (no units), is given in Equation 3-53.

\[
P_{dc} := 1 - \prod_{i} \left(1 - P_d(i) \right) = 0.994 \quad 1 - \left(1 - P_d(1) \right)^N = 0.994
\]

\[
P_{fac} := 1 - \prod_{i} \left(1 - P_{fa}(i) \right) = 10 \times 10^{-6} \quad 1 - \left(1 - P_{fa}(1) \right)^N = 10 \times 10^{-6}
\]

Using Table 3-1 we find the detection threshold associated with \( P_d \) and \( P_{fa} \) is 10.8 dB. Using Table 3-1 we find the detection threshold associated with \( P_{dc} \) and \( P_{fac} \) is 14.0 dB. Thus, by using cumulative detection we can achieve the detection criteria associated with \( P_{dc} \) and \( P_{fac} \) at a lower detection threshold, 10.8 dB instead of 14.0 dB. The lower detection threshold results in an increased detection range, as shown below using Equation 3-58. Note: we used a detection range of one (1) because we only needed to find the change in detection range.

\[
\frac{4}{10} \left( \frac{14.0}{10} \right) \left( \frac{10.8}{10} \right)^{(1)} = 1.2023
\]
Exercise 3-11  A radar system has the following characteristics: time between detection decisions $T_d = 25$ msec, probability of false alarm $P_{fa} = 10^{-6}$, number of range gates $n_{rg} = 300$, number of Doppler filters $n_{df} = 125$, and number of antenna beam positions $n_b = 640$. What is the probability of false alarm when using a two step (alert-confirm) sequential detection approach? What is the average number of false alarms in the first step (alert) of the sequential detection approach, $N_{fa}$? What is the consequence of the second step (confirm) of the sequential detection approach in terms of the number of antenna beam positions (absolute and percent) and additional time (absolute and percent)? [Hint: use the results of Exercise 3-1]

We start by defining the values for all the terms.

$$T_d \equiv 25 \times 10^{-3} \cdot \text{sec} \quad n_{rg} \equiv 300 \quad n_{df} \equiv 125 \quad n_b \equiv 640 \quad P_{fa} \equiv 10^{-6}$$

The probability of false alarm, $P_{fa}$ (no units), when using a two step (alert-confirm) sequential detection approach is given in Equation (3-54).

$$P_{fa}^2 = 1 \times 10^{-12}$$

The average number of false alarms in one time period of interest, $N_{fa}$ (no units), is given in Equation (3-12). First we must compute the number of detection decisions performed over a time period of interest, $N_d$ (no units), using Equation (3-11). Note: we computed the value for $N_d$ in Exercise 3-1.

$$N_d := n_{rg} \cdot n_{df} \cdot n_b = 2.4 \times 10^7$$

$$N_{fa} := N_d \cdot P_{fa} = 24$$

Because the average number of false alarm is 24, the same number of additional beam positions are required as confirmation steps. The percent increase in number of beam positions is

$$\frac{N_{fa}}{n_b} = 3.75\%$$
The 24 additional beam positions require 0.6 seconds additional time, as shown below. The percent increase in time associated with the additional beam positions relative to the scan time, $T_S$ (seconds), given in Equation (3-17), is shown below.

\[
Nfa \cdot T_d = 0.6 \text{s}
\]

\[
T_S := n_b \cdot T_d = 16 \text{s}
\]

\[
\frac{Nfa \cdot T_d}{T_S} = 3.75\% = \frac{24 \cdot (25 \cdot 10^{-3} \text{sec})}{(16 \text{sec})} = 3.75\%
\]
Exercise 3-12  A four attempt binomial detection approach is proposed. The detection threshold is set to provide a $P_d = 0.4$ and $P_{fa} = 10^{-3}$ for each attempt. Calculate the binomial probabilities of detection and false alarm associated with exactly one, two, three, and four detections out of four attempts. Repeat for at least one, two, three, and four detections out of four attempts.

We start by defining the values for all the terms.

$$P_d \equiv 0.4 \quad P_{fa} \equiv 10^{-3}$$

The "M-out-of-N" detection probability using the binomial probability equation, Equation 3-55.

$$P_M := \frac{N!}{M! \cdot (N-M)!} \cdot p^M \cdot (1-p)^{(N-M)}$$

where:

- $P$ = Probability of $M$ detections out of $N$ attempts
- $M$ = Number of required detections
- $N$ = Number of detection attempts
- $p$ = Probability of a single detection attempt being successful

We will use this equation to compute the probabilities for $P_d$ associated with threshold crossings of exactly one, two, three, and four pulses as shown below:

$$P_{d1} := P(1,4,P_d) = 0.3456 \quad P_{d2} := P(2,4,P_d) = 0.3456$$
$$P_{d3} := P(3,4,P_d) = 0.1536 \quad P_{d4} := P(4,4,P_d) = 0.0256$$

While these values are correct, generally a broader view of M-out-of-N detections is taken. As discussed in the text, we sum the exact probabilities to obtain the probability of at least $M$-out-of-$N$ detections, Equation 3-57. The probability of at least $M$-out-of-$N$ detections, $P_{d\_least}$, is shown below:

$$P_{d\_least} := P_{d1} + P_{d2} + P_{d3} + P_{d4} = 0.8704 \quad P_d = 0.4$$

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The probability of detection is increased when either an at least 1-out-of-4 or 2-out-of-4 approach is used.

We will use the same approach to compute the probabilities for $P_{fa}$ associated with threshold crossings of exactly one, two, three, and four false alarms as shown below:

\[ P_{fa1} := P_{1,4} = 3.988 \times 10^{-3} \]
\[ P_{fa2} := P_{2,4} = 5.988 \times 10^{-6} \]
\[ P_{fa3} := P_{3,4} = 3.996 \times 10^{-9} \]
\[ P_{fa4} := P_{4,4} = 1 \times 10^{-12} \]

While these values are correct, generally a broader view of M-out-of-N false alarms is taken. As discussed in the text, we sum the exact probabilities to obtain the probability of at least M-out-of-N false alarms, Equation 3-57. The probability of at least M-out-of-N false alarms, $P_{fa\_least}$, is shown below:

\[ P_{fa\_least1} := P_{fa1} + P_{fa2} + P_{fa3} + P_{fa4} = 3.994 \times 10^{-3} \]
\[ P_{fa} = 1 \times 10^{-3} \]
\[ = 0.003988011996 + 5.9880060000000007E-06 + 3.996E-09 + 1E-12 = 3.994 \times 10^{-3} \]

\[ P_{fa\_least2} := P_{fa2} + P_{fa3} + P_{fa4} = 5.992 \times 10^{-6} \]
\[ = 5.9880060000000007E-06 + 3.996E-09 + 1E-12 = 5.992 \times 10^{-6} \]
The probability of false alarm is decreased when either an at least 2-out-of-4, 3-out-of-4, or 4-out-of-4 approach is used.

\[
P_{fa\_least3} := P_{fa3} + P_{fa4} = 3.997 \times 10^{-9}
\]

\[
= 3.996E-09 + 1E-12 = 3.997 \times 10^{-9}
\]

\[
P_{fa\_least4} := P_{fa4} = 1 \times 10^{-12}
\]
Exercise 3-13  A radar uses a five-attempt binomial detection with a three-out-of-five threshold-crossing approach. The single attempt probability of detection \(P_d = 0.55\), probability of false alarm \(P_{fa} = 10^{-4}\), and associated detection threshold \(S/N = 9.7\) dB (Table 3-1). What is the binomial probability of detection for at least three or more detection attempts? What is the binomial probability of false alarm for at least three or more detection attempts? Use Figure 3-9 or Table 3-1 to determine the detection threshold associated with the binomial probabilities of detection and false alarm after at least three or more detection attempts.

We start by defining the values for all the terms.

\[ P_d = 0.55 \quad P_{fa} = 10^{-4} \]

The "M-out-of-N" detection probability using the binomial probability equation, Equation 3-55.

\[ P(M, N, p) := \frac{N!}{M! \cdot (N-M)!} \cdot p^M \cdot (1-p)^{(N-M)} \]

where:

\[ P \quad = \quad \text{Probability of } M \text{ detections out of } N \text{ attempts} \]
\[ M \quad = \quad \text{Number of required detections} \]
\[ N \quad = \quad \text{Number of detection attempts} \]
\[ p \quad = \quad \text{Probability of a single detection attempt being successful} \]

We will use this equation to compute the probabilities for \(P_d\) associated with threshold crossings of exactly three, four, and five detection attempts as shown below:

\[ P_{d1} := P(3, 5, P_d) = 0.3369 \quad P_{d2} := P(4, 5, P_d) = 0.2059 \quad P_{d3} := P(5, 5, P_d) = 0.0503 \]

We sum the exact probabilities to obtain the probability of at least 3-out-of-5 detections, Equation 3-37. The probability of at least 3-out-of-5 detections, \(P_{d\_least}\), is shown below:

\[ P_{d\_least} := P_{d1} + P_{d2} + P_{d3} = 0.5931 \]

\[ = 0.336909375 + 0.20588906250000003 + 0.050328437500000017 = 0.5931 \]
We will use the same approach to compute the probabilities for $P_{fa}$ associated with threshold crossings of exactly three, four, and five false alarms as shown below:

$$P_{fa1} := P(3, 5, P_{fa}) = 9.998 \times 10^{-12} \quad P_{fa2} := P(4, 5, P_{fa}) = 4.9995 \times 10^{-16} \quad P_{fa3} := P(5, 5, P_{fa}) = 1 \times 10^{-20}$$

We sum the exact probabilities to obtain the probability of at least 3-out-of-5 false alarms. The probability of at least 3-out-of-5 false alarms, $P_{fa\_least}$, is shown below:

$$P_{fa\_least} := P_{fa1} + P_{fa2} + P_{fa3} = 9.9985 \times 10^{-12} \overset{=}{=} 9.9980001E-12 + 4.9995000000000007E-16 + 1.0000000000000001E-20 = 9.9985 \times 10^{-12}$$

Using Table 3-1 we find the detection threshold associated with $P_{d\_least}$ and $P_{fa\_least}$ is 14.3 dB. Thus, by using binomial detection we can achieve the detection criteria associated with $P_{d\_least}$ and $P_{fa\_least}$ at a lower detection threshold, 9.7 dB instead of 14.3 dB.
**Exercise 3-14** When the radar in Exercise 3-13 uses an at least three-out-of-five threshold crossing detection approach its detection range is \(R_{dt} = 100 \text{ km}\). What is the detection range if the radar uses a single threshold crossing detection approach? [Hint: use the results of Exercise 3-13]

In Exercise 3-13 we were given a single attempt detection threshold \(\text{SNR}_{dt1\_dB} = 9.7 \text{ dB}\). We determined a detection threshold \(\text{SNR}_{dt2\_dB} = 14.3 \text{ dB}\) was necessary to give the same overall detection performance, \(P_d\) and \(P_{fa}\), as we would get for the M-out-of-N detection approach.

\[
R_{dt1} = 100 \times 10^3 \cdot \text{m} \quad \text{SNR}_{dt1\_dB} = 9.7 \quad \text{SNR}_{dt2\_dB} = 14.3
\]

Equation 3-43 defines the radar detection range. We write this equation for the two detection ranges and detection thresholds. We can see from these two equations that the vast majority of terms are the same. These same terms can be represented by the constant \(C\). We can then solve for \(C\) in each equation, and then equate the two. These equations are shown below.

\[
R_{dt1} = \frac{4 \left( \frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_I}{(4\pi)^3 \cdot \text{SNR}_{dt1} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R} \right)}{R_{dt1}} = \frac{4 \sqrt{C}}{\text{SNR}_{dt1}}
\]

\[
R_{dt2} = \frac{4 \left( \frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_I}{(4\pi)^3 \cdot \text{SNR}_{dt2} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R} \right)}{R_{dt2}} = \frac{4 \sqrt{C}}{\text{SNR}_{dt2}}
\]

\[
C = R_{dt1}^4 \cdot \text{SNR}_{dt1} = R_{dt2}^4 \cdot \text{SNR}_{dt2}
\]

\[
R_{dt2}^4 = \left( \frac{\text{SNR}_{dt1}}{\text{SNR}_{dt2}} \right) \cdot R_{dt1}^4
\]

Now it’s time to put all the numbers in the equation, which is the same as Equation 3-58. First we convert from decibels to absolute so we can use the values in an algebraic equation.

\[
\text{SNR}_{dt1} := 10 \left( \frac{9.7}{10} \right) = 9.3325
\]

\[
= 10 \left( \frac{9.7}{10} \right) = 9.3325
\]
\[ \text{SNR}_{dt2} := 10 \left( \frac{\text{SNR}_{dt2\_dB}}{10} \right) = 26.9153 \]

\[ R_{dt2} := 4 \sqrt{\frac{\text{SNR}_{dt1}}{\text{SNR}_{dt2}}} \cdot R_{dt1} = 7.6736 \times 10^4 \text{m} \]

\[ R_{dt2} = 76.7361 \text{ km} \]

Even though both detection approaches provide the same overall detection criteria, \( P_d \) and \( P_{fa} \), the resultant detection ranges are different because for a single detection threshold crossing approach the detection threshold is higher than for the binomial approach. Thus, the resultant detection range is shorter than for the binomial approach.
Exercise 3-15  The radar system from Exercise 2-5 has the following additional characteristics: pulse repetition frequency PRF = 500 Hertz, antenna half-power beamwidth $\theta_{3\text{dB}} = 1$ degree, antenna scan rate $\theta_{\dot{\text{dot}}} = 25$ deg/sec, noncoherent pulses, and detection threshold $\text{SNR}_{\text{dt}} = 12$ dB. What is Marcum's optimal noncoherent integration gain, $G_I$ (no units and dB)? What is the target signal-to-noise ratio after integration of multiple pulses, $\text{SNR}_n$ (no units and dB)? What is the resultant radar detection range, $R_{\text{dt}}$ (meters)? [Hint: use the results of Exercise 2-5]

We start by defining the values for all the new terms, using the values from Exercise 2-5, and defining the necessary constants.

$\begin{align*}
\text{PRF} & \equiv 500 \text{ Hz} \\
\theta_{3\text{dB}} & \equiv 1 \text{ deg} \\
\theta_{\dot{\text{dot}}} & \equiv 25 \frac{\text{deg}}{\text{sec}} \\
\text{SNR}_{\text{dt}} & \equiv 12 \\
P_R & \equiv 800 \times 10^3 \text{ W} \\
G_{\text{RT}} & \equiv 38 \\
f_c & \equiv 3 \times 10^9 \text{ Hz} \\
G_{\text{sp}} & \equiv 1 \\
F_{R_{\text{dB}}} & \equiv 6 \\
B_R & \equiv 750 \times 10^3 \text{ Hz} \\
L_{R_{\text{dB}}} & \equiv 13 \\
R_{\text{RT}} & \equiv 150 \times 10^3 \text{ m} \\
S & \equiv 7.9445 \times 10^{-14} \text{ W} \\
N & \equiv 1.1949 \times 10^{-14} \text{ W} \\
\sigma & \equiv 5 \cdot \text{m}^2 \\
c & \equiv 3 \times 10^8 \frac{\text{m}}{\text{sec}} \\
T_0 & \equiv 290 \text{ K} \\
k & \equiv 1.38 \times 10^{-23} \frac{\text{W} \cdot \text{sec}}{\text{K}}
\end{align*}$

Convert values from dB to absolute as we will be using algebraic equations.

$\begin{align*}
G_{\text{RT}} :&= 10^{\left( \frac{38}{10} \right)} = 6.3096 \times 10^3 \\
F_R :&= 10^{\left( \frac{6}{10} \right)} = 3.9811 \\
L_R :&= 10^{\left( \frac{13}{10} \right)} = 19.9526 \\
\text{SNR}_{\text{dt}} :&= 10^{\left( \frac{12}{10} \right)} = 15.8489
\end{align*}$

Marcum's optimal noncoherent integration gain, $G_I$ (no units) and $G_{I_{\text{dB}}}$ (dB), is a function of the number of pulses integrated, as given in Equation (3-37). The number of pulses integrated, $n_p$ (no units), is a function of the integration time and the pulse repetition frequency, as given in Equation
(3-41). For a scanning radar system, the integration time is the target illumination time, $T_{ill}$ (seconds), as given in Equation (3-42). These steps and associated equations are shown below.

$$T_{ill} := \frac{\theta_{3dB}}{\theta_{dot}} = 0.04\, \text{s}$$

$$n_p := T_{ill} \cdot \text{PRF} = 20$$

$$G_I := n_p^{0.76} = 9.745$$

$$G_{I_{-\text{dB}}} := 10 \log(G_I) = 9.8878$$

The target signal-to-noise ratio after integration of multiple pulses, $\text{SNR}_n$ (no units) and $\text{SNR}_{n_{-\text{dB}}}$ (dB), is given in Equation (3-34).

$$\text{SNR}_n := \frac{S \cdot G_I}{N} = 64.7915$$

$$\text{SNR}_{n_{-\text{dB}}} := 10 \log(\text{SNR}_n) = 18.1152$$

By looking back to Exercise 2-5 we can clearly see how much higher the target signal-to-noise ratio is after integration. Which of course, is why radar systems use multiple pulse integration.

The resultant detection range, $R_{dt}$ (meters), is given in Equation (3-43). We need the wavelength, $\lambda$ (meters), and compute it using Equation (2-2).

$$\lambda := \frac{c}{f_c} = 0.1\, \text{m}$$
\[
R_{dt} := \sqrt{\frac{P_R \cdot G_{RT} \cdot \lambda^2 \cdot \sigma \cdot G_{sp} \cdot G_I}{(4\pi)^3 \cdot SNR_{dt} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}} = 2.1329 \times 10^5 \text{m} \quad R_{dt} = 213.289 \text{km}
\]

\[
= \sqrt{\left(800 \times 10^3 \cdot W\right) \left[10 \left(\frac{38}{10}\right)\right]^2 \left(3 \times 10^8 \cdot \frac{m}{\text{sec}}\right)^2 \left(5 \cdot m^2\right) (1) (9.745)} = 2.1329 \times 10^5 \text{m}
\]

We can use the logarithmic form of this equation as well, Equation 3-45. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the radar detection range.

\[
\text{term}_1 := 10 \log \left(\frac{P_R}{1W}\right) + 2 \times G_{RT\_dBi} - 20 \log \left(\frac{f_c}{1Hz}\right) + 10 \log \left(\frac{\sigma}{1m^2}\right) = -47.5218
\]

\[
10 \log \left(\frac{800 \times 10^3 \cdot W}{1W}\right) + 2 \cdot 38 - 20 \log \left(\frac{3 \times 10^9 \cdot \text{Hz}}{1\text{Hz}}\right) + 10 \log \left(\frac{5 \cdot m^2}{1m^2}\right) = -47.5218
\]

\[
\text{term}_2 := 10 \log (G_{sp}) + 10 \log (G_I) - SNR_{dt\_db} - F_{R\_db} = -8.1122
\]

\[
= 10 \cdot \log (1) + 10 \cdot \log (9.74502138972848) - 12 - 6 = -8.1122
\]

\[
\text{term}_3 := -10 \log \left(\frac{B_R}{1Hz}\right) - L_{R\_db} + 340.5434 = 268.7928
\]

\[
= (-10) \cdot \log \left(\frac{750 \cdot 10^3 \cdot \text{Hz}}{(1 \cdot \text{Hz})}\right) - 13 + 340.5434 = 268.7928
\]
\[
\left( \frac{\text{term}_1 + \text{term}_2 + \text{term}_3}{40} \right) \cdot m = 2.1329 \times 10^5 \text{ m}
\]
**Exercise 4-1** You are asked to design an antenna for an infrared radar (wavelength about 20 microns [micrometers]) with a gain of $10^6$ (60 dBi). Assume the antenna efficiency $\rho = 100\%$. What are the dimensions of the aperture? What size would it be in the microwave region, say at S-band ($f_c = 3000$ MHz)?

Note: All the variables for the infrared (IR) part of the problem will use an "_IR" designator, and all variables for the microwave (radio frequency or RF) part of the problem will use an "_RF" designator.

We start by defining the values for all the terms and necessary constants.

\[
\begin{align*}
\lambda_{\text{IR}} &= 20 \times 10^{-6} \text{ m} \\
G &= 10^6 \\
f_{\text{RF}} &= 3000 \times 10^6 \text{ Hz} \\
c &= 3 \times 10^8 \text{ m/sec}
\end{align*}
\]

We start with the equation that related antenna gain with aperture size and wavelength, Equation 4-1. We solve this equation for the effective area, $A_e (m^2)$, and solve, as shown below:

\[
G = \frac{4 \cdot \pi \cdot A_e}{\lambda^2}
\]

\[
A_{e_{\text{IR}}} := \frac{G \cdot \lambda_{\text{IR}}^2}{4 \cdot \pi} = 3.1831 \times 10^{-5} \text{ m}^2
\]

Because the antenna is 100\% efficient, its physical area and effective area are the same. The aperture shape is a square we can compute the length of its side, $a (m)$.

\[
a_{\text{IR}} := \sqrt{A_{e_{\text{IR}}}} = 5.6419 \times 10^{-3} \text{ m}
\]

We compute the aperture size in the microwave region using the same approach. We only need to compute the wavelength, $\lambda_{\text{RF}} (m)$. The whole process is shown below:
\[ \lambda_{\text{RF}} := \frac{c}{f_{\text{RF}}} = 0.1 \text{m} \]

\[ A_{e\text{RF}} := \frac{G \cdot \lambda_{\text{RF}}^2}{4 \cdot \pi} = 795.7747 \text{m}^2 \]

\[ a_{\text{RF}} := \sqrt{A_{e\text{RF}}} = 28.2095 \text{m} \]
Exercise 4-2  A reflector antenna has an area of 100 m². How many half-wave dipole elements would there be in a fully-filled UHF (λ = 1 meter) array of equivalent capability? What would be its area?

We start by defining the values for all the terms.

\[ A_{refl} \equiv 100 \cdot m^2 \quad \lambda \equiv 1 \cdot m \]

The relationship between antenna area, wavelength, and gain for reflector (conventional) antenna is given by Equation 4-1. For array antennas the boresight mainbeam antenna gain relationship is given by Equation 4-23. We will start by computing the gain for the reflector antenna, \( G_{refl} \) (no units), and then solving for the number of elements, \( N \), necessary to achieve the same gain for an array antenna. Because we are using half-wave dipole elements we use Equation 4-26.

\[
G_{refl} := \frac{4 \cdot \pi \cdot A_{refl}}{\lambda^2} = 1.2566 \times 10^3
\]

\[
G_{array} = N \cdot \pi
\]

\[
N := \frac{G_{refl}}{\pi} = 400 \quad \text{elements}
\]

The area of the array, \( A_{array} \) (m²), is determined by equating the two equations above which related the number of elements to antenna gain, area, and wavelength and solving for area, as shown below:

\[
G_{array} = G_{refl} = N \cdot \pi = \frac{4 \pi \cdot A}{\lambda^2}
\]

\[
A_{array} := \frac{\lambda^2 \cdot N}{4} = 100 \cdot m^2
\]
Exercise 4-4 Calculate the mainbeam gain and the azimuth and elevation beamwidths for a radar antenna with the following characteristics: Uniform illumination function, 5 meters wide, 2 meters high, antenna efficiency $\rho = 0.5$, and a frequency $f_c = 3 \text{ GHz}$.

We start by defining the values for all the terms and necessary constants.

$$\text{Width} \equiv 5 \cdot \text{m} \quad \text{Height} \equiv 2 \cdot \text{m} \quad \rho \equiv 0.5 \quad f_c \equiv 3 \times 10^9 \cdot \text{Hz} \quad c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}$$

The mainbeam antenna gain is given by Equation 4-1. The physical area of the antenna product of its width and height. Convert the antenna gain from absolute to decibels relative to isotropic (dBi).

$$G = \frac{4\pi \cdot \rho \cdot A}{\lambda^2} \quad \lambda := \frac{c}{f_c} = 0.1 \text{ m}$$

$$G := \frac{4\pi \cdot \rho \cdot \text{Width} \cdot \text{Height}}{\lambda^2} = 6.2832 \times 10^3$$

$$G_{\text{dBi}} := 10 \log (G) = 37.9818$$

The beamwidth for an antenna with a Uniform illumination function is given by Equation 4-13. The azimuth beamwidth, $\theta_{3\text{dB}_{az}}$ (degrees), is found using this equation and the width of the antenna. The elevation beamwidth, $\theta_{3\text{dB}_{el}}$ (degrees), is found using this equation and the height of the antenna.
\[ \theta_{3dB} = \frac{51 \cdot \lambda}{D} \]

\[ \theta_{3dB\_az} := \frac{51 \cdot \lambda}{\text{Width}} = 1.02 \]

\[ \theta_{3dB\_el} := \frac{51 \cdot \lambda}{\text{Height}} = 2.55 \]
Exercise 4-5  Calculate the normalized gain of an antenna at an angle of 5 degrees for an antenna with a Uniform current distribution, length $D = 3$ meters, and a frequency $f_c = 5$ GHz. If the mainbeam gain $G = 35$ dBi, what is the gain of the antenna at an angle of 5 degrees?

We start by defining the values for all the terms and necessary constants.

$$\theta \equiv 5\,\text{deg} \quad D \equiv 3\,\text{m} \quad f_c \equiv 5 \times 10^9\,\text{Hz} \quad c \equiv 3 \times 10^8\,\frac{\text{m}}{\text{sec}}$$

The equation for normalized antenna gain as a function of angle is given by Equation 4-8. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with $\theta$ above indicates the angle is in degrees (like the "DEG" button on your calculator).

$$\lambda := \frac{c}{f_c} = 0.06\,\text{m}$$

$$G_\theta := \left(\frac{\sin\left(\frac{\pi \cdot D \cdot \sin(\theta)}{\lambda}\right)}{\pi \cdot D \cdot \sin(\theta)}\right)^2 = 4.3394 \times 10^{-3}$$

$$G_{\theta\,\text{dBi}} := 10 \log(G_\theta) = -23.6257$$
The gain of the antenna, $G$ (dBi), at an angle of 5 degrees is given below:

$$G_{\text{dBi}} := 35 + G_{\theta_{\text{dBi}}} = 11.3743$$

$$= 35 + 23.625744526439 = 11.3743$$
Exercise 4-6 A phase steered array has the following characteristics: frequency $f_c = 6 \text{ GHz}$, element spacing $d = \lambda/2$, and phase shift between elements $\Delta \phi = 15^\circ$. What angle is the beam steered to?

We start by defining the values for all the terms and necessary constants.

$$f_c \equiv 6 \times 10^9 \text{ Hz} \quad \Delta \phi \equiv 15 \text{ deg} \quad c \equiv 3 \times 10^8 \frac{\text{m}}{\text{sec}}$$

The beam pointing angle as a function of array characteristics and phase shift between elements is given by Equation 4-36. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with $\Delta \phi$ above indicates the angle is in degrees (like the "DEG" button on your calculator).

$$\Delta \phi = 2\pi \frac{d}{\lambda} \sin(\theta_0) \quad \text{ radians}$$

$$\theta_0 = \sin\left(\frac{\Delta \phi \cdot \lambda}{2\pi \cdot d}\right)$$

$$\lambda := \frac{c}{f_c} = 0.05 \text{ m}$$

$$d := \frac{\lambda}{2} = 0.025 \text{ m}$$

$$\theta_0 := \sin\left(\frac{\Delta \phi \cdot \lambda}{2\pi \cdot d}\right) = 4.7802 \text{ deg}$$
Exercise 4-7  A frequency scan array has a transmit frequency $f_c = 2$ GHz, a frequency excursion $\Delta f = 100$ MHz, element spacing $d = \lambda/2$, and electrical distance between elements $L = 5\lambda$. What is the beam steering limit? What is the wrap-up factor, $L/d$, needed to provide a beam steering limit $\theta_1 = \pm 60^\circ$?

We start by defining the values for all the terms and necessary constants.

$$f_0 \equiv 2 \times 10^9 \cdot \text{Hz} \quad \Delta f \equiv 100 \times 10^6 \cdot \text{Hz} \quad \theta_s \equiv 60 \cdot \text{deg} \quad c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}$$

The beam steering limit, $\theta_1$ (degrees) is given by Equation 4-34. Solve this equation for the beam steering limit, while being careful with radians and degrees. We need the element spacing, $d$ (m), and electrical distance between elements, $L$ (m). We were given relationships for these characteristics relative to a wavelength, $\lambda$ (m). Compute the wavelength, and then $d$ and $L$.

$$\sin(\theta_1) = \frac{L}{2d} \cdot \frac{\Delta f}{f_0} \quad \lambda := \frac{c}{f_0} = 0.15 \text{m}$$

$$d := \frac{\lambda}{2} = 0.075 \text{m}$$

$$L := 5\lambda = 0.75 \text{m}$$

$$\theta_1 := \arcsin\left(\frac{L}{2d} \cdot \frac{\Delta f}{f_0}\right) = 14.4775 \cdot \text{deg}$$
We can solve Equation 4-34 for the wrap-up factor, \( \text{Wrap\_up\_factor} \), (no units), to achieve the required scan limits, \( \theta_s \) (degrees). Note:

Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with \( \theta_s \) above indicates the angle is in degrees (like the "DEG" button on your calculator).

\[
\text{Wrap\_up\_factor} := 2 \cdot \frac{f_0}{\Delta f} \cdot \sin(\theta_s) = 34.641
\]

Note: It is not obvious that \( \Delta \lambda/\lambda_0 \) is the same as \( \Delta f/f_c \) (\( f \) is not the same as \( f_0 \)). We can assume \( \frac{\Delta \lambda}{\Delta \lambda_0} = \frac{\Delta f}{f_c} \) = relative bandwidth. We can show that \( \frac{\Delta \lambda}{\Delta \lambda_0} = \frac{\Delta f}{f_c} \neq \frac{\Delta f}{f_0} \). We start with Equation 4-34.

\[
\sin(\theta_0) = \frac{L}{d} \cdot \left(1 - \frac{f_0}{f_c}\right)
\]

Solving for \( f_0 \) as a function of the frequency that scans to the upper limit of \( \theta_1 \), \( f_u \) (Hz), and as a function of the frequency that scans to the lower limit of \( \theta_1 \), \( f_l \) (Hz), gives the following equations.

\[
f_0 = f_u \cdot \left(1 - \frac{d}{L} \sin(\theta_1)\right) \quad \text{and} \quad f_0 = f_l \cdot \left(1 + \frac{d}{L} \sin(\theta_1)\right)
\]

Subtracting the second equation from the first gives us:

\[
(f_u - f_l) = (f_u + f_l) \left(\frac{d}{L}\right) \sin(\theta_1)
\]
Since \( f_u - f_l = \Delta f \) and \( (f_u + f_l)/2 = f_c \), we have the following equation: This is Equation 4-35.

\[
\frac{\Delta f}{f_c} = 2 \cdot \left( \frac{d}{L} \right) \sin(\theta_1) = \frac{\Delta \lambda}{\lambda_0}
\]

If we use the wavelength equation, \( \lambda = c/f \), we can find

\[
\frac{\Delta \lambda}{\lambda_0} = \frac{\Delta f}{f_0} \cdot \frac{f_0}{f_u} \cdot \frac{f_0}{f_l}
\]

Which indicates that \( \frac{\Delta \lambda}{\lambda_0} \) approximately equals \( \frac{\Delta f}{f_0} \)

This approximation is correct when \( f_0 = \sqrt{f_u \cdot f_l} \)
Exercise 4-8  An array antenna using 3-bit phase shifters and has a beamwidth $\theta_{3\text{dB}} = 2^\circ$. How precisely can the beam be steered?

We start by defining the values for all the terms.

$B \equiv 3 \quad \theta_{3\text{dB}} \equiv 2 \cdot \text{deg}$

The precision to which the beam can be steered is given by Equation 4-37. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with $\theta_{3\text{dB}}$ above indicates the angle is in degrees (like the "DEG" button on your calculator).

$$\Delta \theta_0 := \frac{\pi}{4} \cdot \frac{1}{2^B} \cdot \theta_{3\text{dB}} = 0.1963 \cdot \text{deg}$$  

$$= \frac{\pi}{4} \cdot \left(\frac{1}{2^3}\right) \cdot (2 \cdot \text{deg}) = 0.1963 \cdot \text{deg}$$
Exercise 4-9  Calculate the beamwidth of an electronically scanned array with a beamwidth \( \theta_{3dB} = 2.5^\circ \) when pointed at 0°, when it is steered to 25°, 30°, and 45°.

We start by defining the values for all the terms.

\[ \theta_{3dB} = 2.5 \cdot \text{deg} \]

Use Equation 4-41 to compute the beamwidth as a function of scan angle. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" in the cosine function indicates the angle is in degrees (like the "DEG" button on your calculator).

\[
\theta_{3dB_{25}} := \frac{\theta_{3dB}}{\cos (25 \cdot \text{deg})} = 2.7584 \cdot \text{deg} = \frac{2.5 \cdot \text{deg}}{\cos (25 \cdot \text{deg})} = 2.7584 \cdot \text{deg}
\]

\[
\theta_{3dB_{30}} := \frac{\theta_{3dB}}{\cos (30 \cdot \text{deg})} = 2.8868 \cdot \text{deg} = \frac{2.5 \cdot \text{deg}}{\cos (30 \cdot \text{deg})} = 2.8868 \cdot \text{deg}
\]

\[
\theta_{3dB_{45}} := \frac{\theta_{3dB}}{\cos (45 \cdot \text{deg})} = 3.5355 \cdot \text{deg} = \frac{2.5 \cdot \text{deg}}{\cos (45 \cdot \text{deg})} = 3.5355 \cdot \text{deg}
\]
**Exercise 4-10** Calculate the mainbeam antenna gain (absolute and dBi) for an electronically scanned array with the following characteristics:
number of elements $N = 1000$, element gain $g = 1.5$ dBi, $d/\lambda = 1$, when it is scanned to $25^\circ$, $30^\circ$, and $45^\circ$.

We start by defining the values for all the terms.

\[ N = 1000 \quad g_{\text{dBi}} = 1.5 \]

First we need to convert from decibels to absolute, and then compute the mainbeam antenna gain for reference using Equation 4-23.

\[
g := 10 \left( \frac{g_{\text{dBi}}}{10} \right) = 1.4125 \]

\[
G := N \cdot g = 1.4125 \times 10^3 = 1000 \cdot 1.4125375446227544 = 1.4125 \times 10^3
\]

\[
10 \log (G) = 31.5 \]

The mainbeam antenna gain as a function of scan angle is given by Equation 4-40. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" in the cosine function indicates the angle is in degrees (like the "DEG" button on your calculator). Convert all the mainbeam antenna gain values to decibels relative to isotropic (dBi).

\[
G_{25} := N \cdot g \cdot \cos (25 \cdot \text{deg}) = 1.2802 \times 10^3
\]

\[
= 1000 \cdot 1.4125375446227544 \cdot \cos (25 \cdot \text{deg}) = 1.2802 \times 10^3
\]

\[
10 \log (G_{25}) = 31.0728 \]

\[
G_{30} := N \cdot g \cdot \cos (30 \cdot \text{deg}) = 1.2233 \times 10^3
\]

\[
= 1000 \cdot 1.4125375446227544 \cdot \cos (30 \cdot \text{deg}) = 1.2233 \times 10^3
\]

\[
10 \log (G_{30}) = 30.8753
\]
\[ G_{45} := N \cdot g \cdot \cos(45\, \text{deg}) = 998.8149 \]

\[ = 1000 \cdot 1.4125375446227544 \cdot \cos(45\, \text{deg}) = 998.8149 \]

\[ 10 \log(G_{45}) = 29.9949 \]

\[ = 10 \cdot \log(998.81487648334519) = 29.9949 \]
Exercise 5-1 A radar transmits a $\tau = 0.5$ microsecond pulse at a pulse repetition frequency $\text{PRF} = 300$ Hz. What range resolution and unambiguous range can this radar achieve?

We start by defining the values for all the terms and necessary constants.

$\tau \equiv 0.5 \times 10^{-6} \cdot \text{sec}$  \hspace{1cm} \text{PRF} \equiv 300 \cdot \text{Hz}$  \hspace{1cm} c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}$

The range resolution is given by Equation 5-13.

$$\Delta R := \frac{c \cdot \tau}{2} = 75 \text{m}$$

The unambiguous range is given by Equation 5-20.

$$R_u := \frac{c}{2 \cdot \text{PRF}} = 5 \times 10^5 \text{m}$$

$R_u = 500 \cdot \text{km}$
**Exercise 5-2** What is the dead zone of the radar whose waveform was designed in the preceding question?

First we need the pulse width, $\tau$ (sec), from Exercise 5-1, and define the necessary constants.

$$\tau \equiv 0.5 \times 10^{-6} \cdot \text{sec} \quad \quad c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}$$

As discussed in the Range Measurements Section 5.2.4 the dead zone is the same as the range extent of the pulse. The dead zone is computed below.

$$\text{Dead zone} \equiv \frac{c \cdot \tau}{2} = 75 \text{m}$$

$$= \frac{3 \times 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right) \cdot (0.5 \cdot 10^{-6} \cdot \text{sec})}{2} = 75 \text{m}$$
**Exercise 5-3** A space surveillance radar system has the requirement to resolve spacecraft in range if they are 30 meters apart ($\Delta R = 30 \, \text{m}$) and in Doppler if they are moving at range rates that differ by 15 meters/second ($\Delta R_{\text{dot}} = 15 \, \text{m/sec}$). Design a compressed pulse burst waveform at X-band ($f_c = 10 \, \text{GHz}$) that will meet the requirements.

We start by defining the values for all the terms and necessary constants.

\[
\Delta R = 30 \, \text{m} \quad \Delta R_{\text{dot}} = 15 \cdot \frac{\text{m}}{\text{sec}} \quad f_c = 10 \times 10^9 \, \text{Hz} \quad c = 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}
\]

A compressed pulse is defined by its pulse width, $\tau$ (sec), and modulation bandwidth, $B_{pc}$ (Hz). The relationship between modulation bandwidth and range resolution for a compressed pulse is given by Equation 5-16. This equation is solved for modulation bandwidth as shown below:

\[
\Delta R = \frac{c}{2 \cdot B_{pc}} \quad B_{pc} := \frac{c}{2 \cdot \Delta R} = 5 \times 10^6 \, \text{Hz}
\]

The relationship between range rate resolution and Doppler filter bandwidth is given by Equation 5-30. The relationship between Doppler filter bandwidth and radar integration time (pulse burst duration) is given in Equation 5-29. The wavelength is given in Equation 2-2. Combine these equations and solve for the radar integration time, $T_I$ (seconds), as shown below:

\[
\Delta R_{\text{dot}} = \frac{\Delta f_d \cdot \lambda}{2} \quad \Delta f_d = \frac{1}{T_I} \quad \Delta R_{\text{dot}} = \frac{\lambda}{2 \cdot T_I}
\]

\[
\lambda := \frac{c}{f_c} = 0.03 \, \text{m}
\]

\[
\frac{3 \cdot 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right)}{(10 \cdot 10^9 \, \text{Hz})} = 0.03 \, \text{m}
\]
\[ T_I := \frac{\lambda}{2 \cdot \Delta R_{dot}} = 1 \times 10^{-3} \text{s} \]

\[
\begin{align*}
&= \frac{3 \cdot 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right)}{2 \cdot \left[ 15 \cdot \left( \frac{\text{m}}{\text{sec}} \right) \right]} \cdot \left( 10 \cdot 10^9 \cdot \text{Hz} \right) \\
&= 1 \times 10^{-3} \text{s}
\end{align*}
\]
**Exercise 5-4** An aircraft approach control radar system with a maximum unambiguous range \( R_u = 40 \) kilometers has the requirements to resolve aircraft 30 meters apart in range \((\Delta R = 30 \) m\) and 15 meters/second separation in range rate \((\Delta R_{\text{dot}} = 15 \) m/sec\). Design a pulse burst waveform for this radar at C-band \((f_c = 6 \) GHz\).

We start by defining the values for all the terms and necessary constants.

\[
R_u \equiv 40 \times 10^3 \text{ m} \\
f_c \equiv 6 \times 10^9 \text{ Hz} \\
\Delta R \equiv 30 \text{ m} \\
\Delta R_{\text{dot}} \equiv 15 \frac{\text{m}}{\text{sec}} \\
c \equiv 3 \times 10^8 \frac{\text{m}}{\text{sec}}
\]

We will solve this problem in multiple steps to determine the required pulse burst waveform characteristics: pulse width, PRI - PRF, and pulse burst duration. First, we will solve the range resolution, \(\Delta R\), equation (5-14) for the required pulse length, \(\tau\) (sec).

\[
\Delta R = \frac{c \cdot \tau}{2} \\
\tau := \frac{2 \cdot \Delta R}{c} = 2 \times 10^{-7} \text{ s}
\]

Next we will solve the range ambiguity, \(R_u\), equation (5-20) for the pulse repetition interval, PRI (sec).

\[
R_u = \frac{c \cdot \text{PRI}}{2} = \frac{c}{2 \cdot \text{PRF}}
\]

\[
\text{PRI} := \frac{2 \cdot R_u}{c} = 2.6667 \times 10^{-4} \text{ s}
\]

\[
\text{PRF} := \frac{1}{\text{PRI}} = 3.75 \times 10^3 \text{ Hz}
\]

Finally, we will solve the range rate resolution, \(\Delta R_{\text{dot}}\), equation for the pulse burst duration, \(T_I\) (sec), Equations 5-29 and 5-30. We need the wavelength, \(\lambda\) (meters), so we will calculate it using Equation 2-2.
\[ \lambda := \frac{c}{f_c} = 0.05 \text{m} \]

\[ \Delta R_{\text{dot}} = \frac{\Delta f_d \cdot \lambda}{2} = \frac{\lambda}{2 \cdot T_I} \]

\[ T_I := \frac{\lambda}{2 \cdot \Delta R_{\text{dot}}} = 1.6667 \times 10^{-3} \text{s} \]

\[\begin{align*}
\frac{3 \cdot 10^8 \frac{\text{m}}{\text{sec}}}{(6 \cdot 10^9 \text{Hz})} &= 0.05 \text{m} \\
\frac{3 \cdot 10^8 \frac{\text{m}}{\text{sec}}}{(6 \cdot 10^9 \text{Hz})} &= 1.6667 \times 10^{-3} \text{s}
\end{align*}\]
Exercise 5-5  The waveform of the preceding question is unambiguous in range. The aircraft to be handled have range rates over ±300 meters/second. (a) Design a waveform that is unambiguous in range rate. (b) What is the unambiguous range of this waveform?

First define the values for the terms, including some from Exercise 5-4, and necessary constants.

\[ R_{\text{dotu}} = 300 \cdot \frac{\text{m}}{\text{sec}} \quad f_c = 6 \times 10^9 \cdot \text{Hz} \quad c = 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}} \]

(a) The unambiguous range rate, \( R_{\text{dotu}} \), is a function of the pulse repetition frequency, \( \text{PRF} \) (Hertz), as given in Equation 5-32. We need the wavelength, \( \lambda \) (meters), so we will calculate it using Equation 2-2.

\[
\lambda := \frac{c}{f_c} = 0.05\text{m}
\]

\[
R_{\text{dotu}} = \frac{\text{PRF} \cdot \lambda}{4} = \frac{4 \cdot R_{\text{dotu}}}{\lambda} = 2.4 \times 10^4 \cdot \text{Hz}
\]

(b) We use Equation 5-20 to compute the unambiguous range, \( R_u \) (meters).

\[
R_u := \frac{c}{2 \cdot \text{PRF}} = 6.25 \times 10^3 \text{m}
\]

\[ R_u = 6.25 \cdot \text{km} \]

Exercises 5-4 and 5-5 show how avoiding both range and range rate ambiguities with the same PRF is very difficult. Especially when both long unambiguous range and high unambiguous range rate are required.
Exercise 5-6 A high-precision S-band \((f_c = 3000 \text{ MHz})\) tracker needs to resolve targets 1 km apart in cross range \((\Delta CR = 1 \text{ km})\), 15 meters in range \((\Delta R = 15 \text{ m})\), and 3 meters/second in range rate \((\Delta R_{dot} = 3 \text{ m/sec})\) at ranges of 200 km. (a) What diameter is the antenna if it has a uniform current distribution, \(D\) (meters)? (b) What are the characteristics of the waveform?

We start by defining the values for all the terms and necessary constants.

\[
\begin{align*}
    f_c & = 3000 \times 10^6 \text{ Hz} \\
    \Delta CR & = 1 \times 10^3 \text{ m} \\
    \Delta R & = 15 \text{ m} \\
    \Delta R_{dot} & = 3 \text{ m/sec} \\
    R & = 200 \times 10^3 \text{ m} \\
    c & = 3 \times 10^8 \frac{\text{m}}{\text{sec}}
\end{align*}
\]

We solve part (a) by knowing the following relationships: First, the relationship between the cross range resolution, \(\Delta CR\) (meters), and the range, \(R\) (meters), and angle resolution, \(\Delta \theta\) (radians), as given in Equation 5-39. Second, the angle resolution is equal to the radar antenna half-power beamwidth, \(\theta_{3dB}\) (radians), Equation 5-38. And finally, the radar antenna half-power beamwidth is related to the wavelength, \(\lambda\) (meters), and the antenna diameter, \(D\) (meters), as given in Equation 4-12 for an antenna with a uniform current distribution. The wavelength is computed using Equation 2-2. All the equations and calculations are shown below:

\[
\begin{align*}
    \Delta CR &= R \cdot \Delta \theta \\
    \Delta \theta &= \frac{\Delta CR}{R} = 5 \times 10^{-3} \text{ rad} \\
    \lambda &= \frac{c}{f_c} = 0.1 \text{ m} \\
    \theta_{3dB} &= \frac{0.886 \cdot \lambda}{D} \\
    \theta_{3dB} &= \Delta \theta = 5 \times 10^{-3} \text{ rad} \\
    \theta_{3dB} &= 0.2865 \text{ deg}
\end{align*}
\]
For part (b) we will compute the required radar waveform characteristics: pulse width and pulse burst duration. We need to use the relationships for a pulse burst waveform because we have to achieve both range and range rate measurements. The pulse width, $\tau$ (seconds), is a function of the range resolution, $\Delta R$ (meters), as given in Equation 5-14. The pulse burst duration, $T_I$ (seconds), is a function of the wavelength, $\lambda$ (meters) and range rate resolution, $\Delta R_{dot}$ (meters/second), as given in Equations 5-29 and 5-30. All the equations and calculations are shown below:

$$
\Delta R = \frac{c \cdot \tau}{2}
$$

$$
\tau := \frac{2 \cdot \Delta R}{c} = 1 \times 10^{-7} \text{ s}
$$

$$
\Delta R_{dot} = \frac{f_d \cdot \lambda}{2} = \frac{\lambda}{2 \cdot T_I}
$$

$$
T_I := \frac{\lambda}{2 \cdot \Delta R_{dot}} = 0.0167 \text{ s}
$$
Exercise 5-7  A radar system has a frequency modulated (FM) chirp waveform with a bandwidth of $B_{pc} = 100 \text{ MHz}$ over a transmitted pulse width $\tau = 200 \text{ microseconds}$.  (a) What is the time-bandwidth product?  (b) What is the range resolution, $\Delta R_{pc}$ (meters)?

We start by defining the values for all the terms and necessary constants.

\[
B_{pc} \equiv 100 \times 10^6 \cdot \text{Hz} \quad \tau \equiv 200 \times 10^{-6} \cdot \text{sec} \quad c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}
\]

(a) The time-bandwidth product is given in Equation 5-19. The results are shown below:

\[
\tau \cdot B_{pc} = 2 \times 10^4 = 200 \cdot 10^{-6} \cdot \text{sec} \cdot (100 \cdot 10^6 \cdot \text{Hz}) = 2 \times 10^4
\]

To solve part b) we use the range resolution, $\Delta R_{pc}$ (m), with pulse compression, Equation 5-16.

\[
\Delta R_{pc} := \frac{c}{2 \cdot B_{pc}} = 1.5 \text{ m} = \frac{3 \cdot 10^8 \cdot \left(\frac{\text{m}}{\text{sec}}\right)}{2 \cdot (100 \cdot 10^6 \cdot \text{Hz})} = 1.5 \text{ m}
\]
**Exercise 5-8** Show the signal-to-noise ratio (S/N) with pulse compression processing equals the S/N without. Assume frequency modulation (FM) or "chirp" pulse compression.

We start with a few definitions: \( S \) = received target signal power, \( N \) = receiver thermal noise power, \( S_{pc} \) = received target signal power after pulse compression signal processing, \( N_{pc} \) = receiver thermal noise power with pulse compression signal processing, \( \tau \) = transmitted pulse width, \( B_{pc} \) = pulse compression modulation bandwidth, \( k \) = Boltzmann's constant, \( F_R \) = receiver noise figure, \( T_0 \) = receiver standard temperature, and \( B_R \) = receiver bandwidth matched to the transmitted pulse width.

The receiver thermal noise power, \( N \), is given by Equation 2-12. When pulse compression is being used, the receiver bandwidth has to be at least the pulse compression modulation bandwidth. Thus, the receiver thermal noise power when using pulse compression, \( N_{pc} \), is given below based on Equations 2-16 and 5-18.

\[
N = k \cdot F_R \cdot T_0 \cdot B_R \\
N_{pc} = k \cdot F_R \cdot T_0 \cdot B_{pc}
\]

The received target signal power out of the pulse compression processor, \( S_{pc} \), is the input target signal power multiplied by the time-bandwidth product (Equation 5-19), or pulse compression ratio (PCR).

\[
S = S_{pc} = S \cdot \text{PCR} = S \cdot (\tau \cdot B_{pc})
\]

Now form the signal-to-noise ratio without pulse compression signal processing, \( \text{SNR} \), and the signal-to-noise ratio with pulse compression signal processing, \( \text{SNR}_{pc} \). We then simplify the \( \text{SNR}_{pc} \) equation.

\[
\text{SNR} = \frac{S}{k \cdot F_R \cdot T_0 \cdot B_R} \\
\text{SNR}_{pc} = \frac{S \cdot (\tau \cdot B_{pc})}{k \cdot F_R \cdot T_0 \cdot B_{pc}} = \frac{S \cdot \tau}{k \cdot F_R \cdot T_0}
\]

For a matched filter receiver we know the receiver bandwidth is defined by Equation 2-15. The numerator is now the receiver thermal noise power. Thus, we are left with \( \text{SNR}_{pc} = \text{SNR} \), and conservation of energy holds true.

\[
B_R = \frac{1}{\tau} \\
\text{SNR}_{pc} = \frac{S}{k \cdot F_R \cdot T_0 \cdot B_R} = \text{SNR}
\]
**Exercise 5-9** A radar system with a transmitted pulse width of $\tau = 2 \, \mu\text{sec}$ is required to have a range resolution $\Delta R = 5$ meters. Can the radar system provide the required range resolution with its transmitted pulse width, why or why not? If not, phase modulation pulse compression will be used. How many phase coded segments or chips are necessary to provide the required range resolution, $N\phi$ (no units)?

Start by defining the values for all the terms and necessary constants.

\[
\Delta R \equiv 5 \cdot \text{m} \\
\tau \equiv 2 \times 10^{-6} \cdot \text{sec} \\
c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}
\]

We first compute the range resolution, $\Delta R$ (meters), based on the transmitted pulse width, as given in Equation 5-14.

\[
\frac{c \cdot \tau}{2} = 300 \text{ m}
\]

We can see from this result that the radar system cannot provide the required range resolution with its transmitted pulse width. Thus, we need to determine how many phase coded segments or chips are necessary to provide the required range resolution. We will do this in multiple steps. First, the range resolution with pulse compression is a function of the pulse compression bandwidth, $B_{pc}$, as given in Equation 5-16. The bandwidth of a phase modulated pulse compression waveform is a function of the transmitted pulse width and the number of chips, $N\phi$ (no units), as given in Equation 5-17. We combine these two equations and solve for the number of chips. All these equations and calculations are shown below:

\[
\Delta R_{pc} = \frac{c}{2 \cdot B_{pc}} \\
B_{pc} = \frac{N\phi}{\tau}
\]

\[
\Delta R_{pc} = \frac{c}{2 \cdot \frac{N\phi}{\tau}} \\
N\phi := \frac{c \cdot \tau}{2 \cdot \Delta R} = 60
\]

\[
= \frac{3 \cdot 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right) \cdot (2 \cdot 10^{-6} \cdot \text{sec})}{2 \cdot (5 \cdot \text{m})} = 60
\]
Exercise 5-10 A radar system has the following characteristics: pulse repetition frequency \( PRF = 1000 \text{ Hz} \), transmitted pulse width \( \tau = 1 \mu\text{sec} \), pulse burst duration \( T_I = 40 \text{ msec} \), azimuth beamwidth \( \theta_{3\text{dB}} = 2 \text{ degrees} \), and elevation beamwidth \( \phi_{3\text{dB}} = 4 \text{ degrees} \). The radar system has an angular scan coverage in the azimuth plane of \( \theta_S = 360 \text{ degrees} \) and in the elevation plane of \( \phi_S = 8 \text{ degrees} \). Determine the number of range gates, \( n_{rg} \) (no units), the number of Doppler filters, \( n_{df} \) (no units), and the number of antenna beam positions, \( n_b \) (no units).

We start by defining the values for all the terms and the necessary constants.

\[
\begin{align*}
PRF &\equiv 1000 \cdot \text{Hz} \\
\tau &\equiv 1 \times 10^{-6} \cdot \text{sec} \\
T_I &\equiv 40 \times 10^{-3} \cdot \text{sec} \\
\theta_{3\text{dB}} &\equiv 2 \cdot \text{deg} \\
\phi_{3\text{dB}} &\equiv 4 \cdot \text{deg} \\
\theta_S &\equiv 360 \cdot \text{deg} \\
\phi_S &\equiv 8 \cdot \text{deg} \\
c &\equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}
\end{align*}
\]

The number of range gates, \( n_{rg} \) (no units), is a function of the unambiguous range, \( R_u \) (meters), and range resolution, \( \Delta R \) (meters), as given in Equation 5-42. The unambiguous range is given in Equation 5-20 and the range resolution is given in Equation 5-14. These equations and the associated calculations are shown below.

\[
\begin{align*}
R_u :&= \frac{c}{2 \cdot PRF} = 1.5 \times 10^5 \text{m} \\
\Delta R :&= \frac{c \cdot \tau}{2} = 150 \text{m}
\end{align*}
\]

\[
\begin{align*}
n_{rg} :&= \frac{R_u}{\Delta R} = 1000
\end{align*}
\]

The number of Doppler filters, \( n_{df} \) (no units), is a function of the pulse repetition frequency, \( PRF \) (Hertz), and Doppler filter bandwidth, \( \Delta f_d \) (Hertz), as given in Equation 5-43. The Doppler resolution is given in Equation 5-29. These equations and the associated calculations are shown below.
\[ \Delta f_d := \frac{1}{T_i} = 25 \cdot \text{Hz} \]

\[ = \frac{1}{(40 \cdot 10^{-3} \cdot \text{sec})} = 25 \cdot \text{Hz} \]

\[ n_{df} := \frac{\text{PRF}}{\Delta f_d} = 40 \]

\[ = \frac{1000 \cdot \text{Hz}}{(25 \cdot \text{Hz})} = 40 \]

The number of antenna beam positions, \( n_b \) (no units), is a function of the azimuth and elevation scan coverages and half-power beamwidths, as given in Equation 5-44.

\[ n_b := \frac{\theta_S \cdot \phi_S}{\theta_{3\text{dB}} \cdot \phi_{3\text{dB}}} = 360 \]

\[ = \frac{360 \cdot \text{deg} \cdot (8 \cdot \text{deg})}{[2 \cdot \text{deg} \cdot (4 \cdot \text{deg})]} = 360 \]
Exercise 5-11  (a) Design a pulse-Doppler (pulse burst) waveform for a transmitted frequency $f_c = 3000 \text{ MHz}$ that is unambiguous in range rate for aircraft flying at $R_{dotu} = ±900 \text{ meters/second}$, can resolve aircraft separated by 30 meters in range ($\Delta R$) and 15 meters/second in range rate ($\Delta R_{dot}$).

(b) What is the associated unambiguous range, $R_u$ (meters)?

(c) Will the waveform from part (a) support the required measurement performance at UHF ($\lambda = 0.6 \text{ meters}$)?

We start by defining the range rate ambiguity, $R_{dotu} (\text{m/sec})$, range resolution, $\Delta R (\text{m})$, and range rate resolution, $\Delta R_{dot} (\text{m/sec})$, the UHF wavelength, $\lambda_{UHF} (\text{m})$. Also define the necessary constants and compute the wavelength using Equation 2-2.

$$R_{dotu} \equiv 900 \cdot \frac{\text{m}}{\text{sec}} \quad \Delta R \equiv 30 \cdot \text{m} \quad f_c \equiv 3000 \times 10^6 \cdot \text{Hz} \quad \Delta R_{dot} \equiv 15 \cdot \frac{\text{m}}{\text{sec}}$$

$$\lambda_{UHF} \equiv 0.6 \cdot \text{m} \quad c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}$$

$$\lambda := \frac{c}{f_c} = 0.1 \text{ m}$$

(a) We will compute the necessary pulse-Doppler (pulse burst) waveform characteristics. The pulse width, $\tau (\text{sec})$, is computed based on the range resolution, $\Delta R (\text{m})$, Equation 5-15. The duration of the pulse burst, $T_I (\text{sec})$, is computed based on the range rate resolution, $\Delta R_{dot} (\text{m/sec})$, Equation 5-30. The pulse repetition frequency, PRF (Hz), is computed based on the range rate ambiguity, $R_{dotu} (\text{m/sec})$, Equation 5-32. All the equations and values are shown below:

$$\Delta R = \frac{c \cdot \tau}{2} \quad \tau := \frac{2 \cdot \Delta R}{c} = 2 \times 10^{-7} \text{ s}$$

$$= \frac{2 \cdot (30 \cdot \text{m})}{3 \cdot 10^8 \cdot \frac{\text{m}}{\text{sec}}} = 2 \times 10^{-7} \text{ s}$$
\[ \Delta R_{\text{dot}} = \frac{\lambda}{2 \cdot T_I} \]

\[ T_I := \frac{\lambda}{2 \cdot \Delta R_{\text{dot}}} = 3.333 \times 10^{-3} \text{s} \]

\[ R_{\text{dot}_u} = \frac{\text{PRF} \cdot \lambda}{4} \]

\[ \text{PRF} := \frac{4 \cdot R_{\text{dot}_u}}{\lambda} = 3.6 \times 10^4 \text{ Hz} \]

(b) The range ambiguity, \( R_u \) (m), is computed based on the PRF, Equation 5-20, as shown below:

\[ R_u := \frac{c}{2 \cdot \text{PRF}} = 4.1667 \times 10^3 \text{ m} \]

\[ = \frac{3 \times 10^8 \left( \frac{\text{m}}{\text{sec}} \right)}{2 \cdot (3.6 \cdot 10^4 \text{ Hz})} = 4.1667 \times 10^3 \text{ m} \]

(c) In this part of the exercise we will determine the radar measurement performance we obtain using the waveform computed in (a), only with \( \lambda_{\text{UHF}} \). The equations and values are shown below:

\[ \Delta R_{\text{UHF}} := \frac{c \cdot \tau}{2} = 30 \text{ m} \]

\[ \Delta R_{\text{dot, UHF}} := \frac{\lambda_{\text{UHF}}}{2 \cdot T_I} = 90 \frac{\text{m}}{\text{s}} \]

\[ = \frac{0.6 \text{ m}}{2 \cdot (3.3333 \cdot 10^{-3} \text{ s})} = 90 \cdot 0.009 \frac{\text{m}}{\text{s}} \]
As can be seen from these values, the waveform can meet the required range resolution, does not meet the required range rate resolution, and exceeds the required range rate ambiguity with $\lambda_{UHF}$. This is no surprise, because any range rate / Doppler-related value is a function of $\lambda$.

\[
\Delta R = 30 \text{ m} \quad \Delta R_{\text{dot}} = 15 \frac{\text{m}}{\text{s}} \quad R_{\text{dotu}} = 900 \frac{\text{m}}{\text{s}}
\]
Exercise 6-1  You are estimating the RCS of a particular type of aircraft that will frequently fly through the coverage of a surveillance radar you are designing. You find that it has an L-band antenna of area, A, that will be directly in the radar line of sight much of the time. Your radar is at X-band. Assume the antenna is a flat plate. (a) What will the RCS of that antenna be? (b) if it were 50 percent efficient and matched to your radar, what would its RCS be?

Using either Equation 6-5 or Table 6-1 we find the RCS of a flat plate is given by the following equation:

\[ \sigma = \frac{4 \cdot \pi \cdot A^2}{\lambda^2} \]

If we use the wavelength of our X-band radar in this equation we can compute the RCS of the flat plate antenna.

For part (b), if the antenna is 50% efficient its effective area is 50% of its physical area. Thus, the RCS of the 50% efficient antenna is given by the following equation:

\[ \sigma = \frac{4 \cdot \pi \cdot A_{\text{eff}}^2}{\lambda^2} = \frac{4 \cdot \pi \cdot (0.5 \cdot A)^2}{\lambda^2} = \frac{\pi \cdot A^2}{\lambda^2} \]
Exercise 6-2  You are supervising the designers of a radar for finding and mapping rainfall. In the region where it will operate, raindrops range in diameter from 2.5 millimeters to 13 millimeters. What frequencies should be used?

Because we are finding and mapping rainfall, we need to determine the frequency that will produce the maximum RCS of the raindrops. Using Figure 6-2 we see that the maximum RCS of a sphere of radius \( a \) is at the transition from the Rayleigh region to the Resonance region. The frequency (wavelength) associated with this transition is shown in the following relationship (the x-axis of Figure 6-2):

\[
2 \cdot \pi \cdot \frac{a}{\lambda} = 1
\]

We can solve this equation for \( \lambda \) and compute wavelengths associated with the range of raindrop diameters. Note: the RCS is given as a function of radius, \( a \). The wavelengths are then converted to frequency. We will solve this using meters, so some unit conversions will have to be performed. All this is shown below:

\[
\lambda = 2 \cdot \pi \cdot a
\]

\[
a_1 = \frac{2.5 \cdot \text{mm}}{2}
\]

\[
a_2 = \frac{13 \cdot \text{mm}}{2}
\]

\[
\lambda_1 := 2 \cdot \pi \cdot a_1 = 7.854 \times 10^{-3} \text{m}
\]

\[
= 2 \cdot \pi \cdot \left( \frac{2.5 \cdot \text{mm}}{2} \right) = 7.854 \times 10^{-3} \text{m}
\]

\[
\lambda_2 := 2 \cdot \pi \cdot a_2 = 0.0408 \text{m}
\]

\[
= 2 \cdot \pi \cdot \left( \frac{13 \cdot \text{mm}}{2} \right) = 0.0408 \text{m}
\]

\[
f_1 := \frac{c}{\lambda_1} = 3.8197 \times 10^{10} \text{Hz}
\]

\[
= \frac{3 \cdot 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right)}{7.854 \cdot 10^{-3} \cdot \text{m}} = 3.8197 \times 10^{10} \text{Hz}
\]

\[
f_2 := \frac{c}{\lambda_2} = 7.3456 \times 10^9 \text{Hz}
\]

\[
= \frac{3 \cdot 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right)}{0.0408 \cdot \text{m}} = 7.3529 \times 10^9 \text{Hz}
\]
We will use the average of the two frequencies, as shown below.

\[
\frac{f_1 + f_2}{2} = 2.2771 \times 10^{10} \text{ Hz}
\]

\[
= \frac{3.8197 \times 10^{10} \text{ Hz} + 7.3456 \times 10^{9} \text{ Hz}}{2} = 2.2771 \times 10^{10} \text{ Hz}
\]
**Exercise 6-3** An aircraft of 30 meter wingspan is flying toward a radar of frequency $f_c = 900$ MHz. Over what changes in aspect angle would the aircraft have to move in order to make a measurement of wingspan?

We start by defining the values for all the terms and necessary constants.

\[
D \equiv 30 \cdot \text{m} \\
\lambda := \frac{c}{f_c} = 0.3333 \text{m}
\]

\[
\lambda := \frac{c}{f_c} = \frac{3 \times 10^8 \text{m}}{900 \times 10^6 \text{Hz}} = 0.3333 \text{m}
\]

This exercise is based on the discussion of Radar Signatures in Section 6.8. By observing the lobe pattern one can infer target characteristics. We will assume the wing of the aircraft is shaped like a long thin rod (drawing in the upper left of Figure 6-6). We can observe the "beamwidth", $\theta$ (radians), Equation 6-28, of the lobe pattern and then compute the length of the rod, D (m). The equations and values are shown below:

\[
\theta := \frac{\lambda}{2 \cdot D} = 5.556 \times 10^{-3} \text{ rad}
\]

\[
\theta := \frac{\lambda}{2 \cdot D} = \frac{3 \times 10^8 \text{m}}{2 \cdot (30 \text{m})} = 5.556 \times 10^{-3} \text{ rad}
\]

\[
\theta = 0.3183 \text{ deg}
\]

\[
\theta = 0.005555555555555549 \times \frac{180 \text{ deg}}{(\pi \text{ rad})} = 0.3183 \text{ deg}
\]
Exercise 6-4  A sailboat uses a dihedral corner reflector with a dimension \( a = 0.3 \) meters to provide a high radar cross section to marine radar systems. A marine radar system with a carrier frequency \( f_c = 10 \) GHz can detect the dihedral corner reflector at a range \( R_{dt} = 10 \) km. A sign at the marine supply store states replacing a dihedral corner reflector with a trihedral corner reflector of the same size results in a 50% increase in radar cross section and radar detection range. Determine if these statements are true or false.

We start by defining the values for all the terms and necessary constants.

\[
a \equiv 0.3 \cdot \text{m} \quad f_c \equiv 10 \times 10^9 \cdot \text{Hz} \quad R_{dt1} \equiv 10 \times 10^3 \cdot \text{m} \quad c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}
\]

We will start by computing the radar cross section (RCS) of the dihedral, \( \sigma_d \) (m\(^2\)), and trihedral, \( \sigma_t \) (m\(^2\)), corner reflectors using Equations 6-8 and 6-9, respectively. We need the wavelength, \( \lambda \) (meters), for these equations and compute it using Equation 2-2.

\[
\lambda := \frac{c}{f_c} = 0.03 \text{m}
\]

\[
\sigma_d := \frac{8\pi \cdot a^4}{\lambda^2} = 226.1947 \text{m}^2
\]

\[
\sigma_t := \frac{12\pi \cdot a^4}{\lambda^2} = 339.292 \text{m}^2
\]

As shown in the percentage difference calculation below, the trihedral corner reflector results in a 50% increase in RCS. So that part of the
The change in radar detection range based on the change in target RCS is given in Equation 6-14. If you have really good eyes you can see the same result using Figure 6-4.

\[ R_{dt2} = 11.0668 \text{ km} \]

As shown in the percentage difference calculation below, the trihedral corner reflector results in much less than a 50% increase in detection range. So that part of the statement on the sign is false. Of course, we know already knew from Equation 6-14 and Figure 6-4 a 50% increase in RCS does not result in a 50% increase in detection range.

\[ \frac{R_{dt2} - R_{dt1}}{R_{dt1}} = 10.6682\% \]
**Exercise 6-5** 100,000 aluminum chaff dipoles might weigh a few pounds at L-band \( (\lambda = 0.3 \text{ meters}) \). What would be the cloud's average RCS?

First we define the number of chaff dipoles, \( N \), and the wavelength.

\[
N \equiv 100000 \quad \lambda \equiv 0.3 \cdot \text{m}
\]

The average RCS of a chaff cloud, \( \sigma_{\text{bar}} \) \( (\text{m}^2) \), is given by Equation 6-24.

\[
\sigma_{\text{bar}} := 0.15 \cdot N \cdot \lambda^2 = 1.35 \times 10^3 \text{m}^2
\]

\[
= 0.15 \cdot 100000 \cdot (0.3 \cdot \text{m})^2 = 1.35 \times 10^3 \text{m}^2
\]
**Exercise 6-6** A radar system has the following characteristics: half-power beamwidth $\theta_{3dB} = 1.5$ degrees and transmitted pulse width $\tau = 1$ μsec. The radar system is attempting to detect a target at a range $R = 15$ km in clutter with a reflectivity $\sigma_0 = -20$ dBsm and a grazing angle of $\alpha = 2$ degrees. What is the clutter radar cross section, $\sigma_c$ (m$^2$ and dBsm)?

Note: The units on the clutter reflectivity is technically just dB. Although it is often given in dBsm to preserve the absolute units, in this case m$^2$/m$^2$.

We start by defining the values for all the terms and necessary constants.

\[
\begin{align*}
\theta_{3dB} &\equiv 1.5 \cdot \text{deg} & \tau &\equiv 1 \times 10^{-6} \cdot \text{sec} & R &\equiv 15 \times 10^3 \cdot \text{m} & \sigma_{0_{dB}} &\equiv -20 & \alpha &\equiv 2 \cdot \text{deg} & c &\equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}
\end{align*}
\]

First we convert the clutter reflectivity from decibels to absolute, and include it units (m$^2$/m$^2$).

\[
\sigma_0 := 10 \left(\frac{\sigma_{0_{dB}}}{10}\right) = 10 \left(\frac{-20}{10}\right) = 0.01 \cdot \frac{\text{m}^2}{\text{m}^2}
\]

The clutter radar cross section, $\sigma_c$ (m$^2$) and $\sigma_{c_{dBsm}}$ (dBsm), is a function of the clutter reflectivity and area of the clutter intercepted by the radar resolution cell, $A_c$ (m$^2$), as given in Equation 6-25. The area of the clutter is given in Equation 6-26. Note: in this equation $\theta_{3dB}$ is in radians, thus we need to convert the provided value from degrees to radians, thus the $\pi / 180$ deg in the equation. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with $\alpha$ above indicates the angle is in degrees (like the "DEG" button on your calculator).

\[
A_c := R \cdot \left(\theta_{3dB} \times \frac{\pi}{180 \cdot \text{deg}}\right) \cdot \left(\frac{c \cdot \tau}{2}\right) \cdot \frac{1}{\cos (\alpha)} = 5.8941 \times 10^4 \text{ m}^2
\]
\[(15 \times 10^3 \text{ m}) \cdot (1.5 \text{ deg} \times \frac{\pi}{180 \text{ deg}}) \cdot \left(\frac{3 \times 10^8 \text{ m}}{\text{sec}} \cdot (1 \times 10^{-6} \text{ sec}) \right) \cdot \frac{1}{\cos (2 \text{ deg})} = 5.8941 \times 10^4 \text{ m}^2\]

\[\sigma_c := \sigma_0 \cdot A_c = 589.4077 \text{ m}^2 \]

\[\sigma_{c_{\text{dBsm}}} := 10 \log \left(\frac{\sigma_c}{1 \text{ m}^2}\right) = 27.7042\]

\[\sigma_{c_{\text{dBsm}}} = 0.01 \cdot (5.8941 \times 10^4 \text{ m}^2) = 589.41 \text{ m}^2\]

\[\sigma_{c_{\text{dBsm}}} = 10 \cdot \log \left(\frac{589.4077 \text{ m}^2}{1 \text{ m}^2}\right) = 27.7042\]
Exercise 7-1 A radar system has the following characteristics: half power beamwidth $\theta_{3\text{dB}} = 2$ degrees, transmitted pulse width $\tau = 1$ microsecond, transmitted carrier frequency $f_c = 3$ GHz, pulse repetition frequency $\text{PRF} = 500$ Hz, and number of pulses integrated $n_p = 40$. (a) What is the signal-to-clutter ratio for the radar system trying to see a one square meter RCS target at 10 kilometers range in clutter having a clutter reflectivity $\sigma_0$ of -30 dBsm and a 1 degree grazing angle? (b) What is the moving target indicator improvement factor, $I_{\text{MTI}}$ (no units and dB), for a two delay $N = 2$ MTI when the standard deviation of the clutter source range rate $\sigma_{\text{Rdots}} = 1$ m/sec and standard deviation of the radar instabilities power spectral density $\sigma_i = 5$ Hz? (c) What is the output signal-to-clutter ratio after the MTI, $\text{SCR}_{\text{out}}$ (no units and dB)?

Note: The units on the clutter reflectivity is technically just dB. Although it is often given in dBsm to preserve the absolute units, in this case m²/m².

We start by defining the values for all the terms and necessary constants. We also convert the clutter reflectivity from "dBsm" to m²/m².

$$
\theta_{3\text{dB}} \equiv 2 \cdot \text{deg} \\
\tau \equiv 1 \times 10^{-6} \cdot \text{sec} \\
f_c \equiv 3 \times 10^9 \cdot \text{Hz} \\
\text{PRF} \equiv 500 \cdot \text{Hz} \\
n_p \equiv 40 \\
\sigma \equiv 1 \cdot \text{m}^2 \\
R \equiv 10 \times 10^3 \cdot \text{m} \\
\sigma_{0 \text{dBsm}} \equiv -30 \\
\alpha \equiv 1 \cdot \text{deg} \\
N \equiv 2 \\
\sigma_{\text{Rdots}} \equiv 1 \cdot \frac{\text{m}}{\text{sec}} \\
\sigma_i \equiv 5 \cdot \text{Hz} \\

c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}} \\

\sigma_0 \equiv 10 \left( \frac{\sigma_{0 \text{dBsm}}}{10} \right) = 1 \times 10^{-3} \cdot \frac{\text{m}^2}{\text{m}^2} = 1 \times 10^{-3} \cdot \frac{\text{m}^2}{\text{m}^2} = 10 \left[ \frac{(-30)}{10} \right] = 1 \times 10^{-3} \cdot \frac{\text{m}^2}{\text{m}^2}
$$

(a) The signal-to-clutter ratio, SCR (no units) and SCR dB (dB), is computed using Equation 7-1. The SCR is converted to decibels as well, SCR dB (dB). Note: in this equation $\theta_{3\text{dB}}$ is in radians, thus we need to convert the provided value from degrees to radians ($\pi / 180$ deg). Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with $\alpha$ indicates the angle is in degrees (like the "DEG" button on your calculator).

$$
\text{SCR} := \frac{2 \cdot \cos (\alpha) \cdot \sigma}{\sigma_0 \cdot R \cdot \left( \theta_{3\text{dB}} \times \frac{\pi}{180 \cdot \text{deg}} \right) \cdot c \cdot \tau} = 0.0191
$$
2 \cdot \cos (1 \cdot \text{deg}) \cdot (1 \cdot \text{m}^2) \\ \left(1 \times 10^{-3} \frac{\text{m}^2}{\text{m}^2}\right) \cdot (10 \times 10^{-3} \cdot \text{m}) \cdot \left(2 \cdot \text{deg} \times \frac{\pi}{180 \cdot \text{deg}}\right) \cdot \left(3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}\right) \cdot (1 \times 10^{-6} \cdot \text{sec}) = 0.0191

\text{SCR}_\text{dB} := 10 \log (\text{SCR}) = -17.1906 = 10 \cdot \log (0.019095684362781383) = -17.1906

(b) The MIT improvement factor, \( I_{\text{MIT}} \) (no units), is a function of MTI characteristics and the clutter power spectral density (PSD), as given in Equation 7-7. The standard deviation of the clutter PSD, \( \sigma_{\text{Cpsd}} \) (Hz), is given in Equation 7-10. The standard deviation of the clutter PSD is a function of the standard deviation of the radar instabilities, \( \sigma_i \) (Hertz), given above, the standard deviation of the clutter source PSD, \( \sigma_s \) (Hertz), Equation 7-8, and the standard deviation of the antenna modulation PSD, \( \sigma_a \) (Hertz), Equation 7-9. We also need the wavelength, \( \lambda \) (meters), Equation 2-2. These equations and calculations are shown below.

\[ \lambda := \frac{c}{f_c} = 0.1 \text{m} \]

\[ \sigma_s := \frac{2 \cdot \sigma_{\text{Rdots}}}{\lambda} = 20 \cdot \text{Hz} \]

\[ \sigma_a := \frac{0.265 \cdot \text{PRF}}{n_p} = 3.3125 \cdot \text{Hz} \]

\[ \sigma_{\text{Cpsd}} := \sqrt{\sigma_i^2 + \sigma_s^2 + \sigma_a^2} = 20.88 \cdot \text{Hz} \]

\[ = \sqrt{(5 \cdot \text{Hz})^2 + (20 \cdot \text{Hz})^2 + (3.3125 \cdot \text{Hz})^2} = 20.88 \cdot \text{Hz} \]
\[ I_{\text{IFMTI}} := \frac{2^N}{N!} \left[ \frac{1}{2\pi \left( \frac{\sigma_{\text{Cpsd}}}{\text{PRF}} \right)} \right]^{2N} = 421.9607 \]

\[ = \frac{2^2}{2!} \left[ \frac{1}{2 \cdot \pi \cdot \left( \frac{20.88\text{ Hz}}{500\text{ Hz}} \right)} \right]^{(2 \cdot 2)} = 421.9574 \]

\[ 10 \log (I_{\text{IFMTI}}) = 26.2527 \]

\[ = 10 \cdot \log (421.96073129059107) = 26.2527 \]

(c) The output signal-to-clutter ratio, \( \text{SCR}_{\text{out}} \) (no units), is computed using Equation 7-6 as shown below.

\[ \text{SCR}_{\text{out}} := I_{\text{IFMTI}} \cdot \text{SCR} = 8.0576 \]

\[ 10 \log (\text{SCR}_{\text{out}}) = 9.0621 \]

\[ = 10 \cdot \log (8.0576289382135364) = 9.0621 \]
Exercise 7-2  Derive the equation for signal-to-clutter ratio (S/C), Equation 7-1

Start with the equation for the received target signal power, Equation 2-11. The received clutter signal power is computed using the same equation, only the target radar cross section (RCS), $\sigma$, is replaced by the clutter RCS, $\sigma_c$. This received clutter signal power is the competing clutter, i.e., at the same range and angle (and thus, antenna gain) as the target. Also we will assume the clutter source is essentially in the same angular position as the target so the same antenna gain is used for both the target and clutter. We can now define the S/C. All the terms cancel out except the target RCS and clutter RCS. All these equations are shown below.

\[
S = \frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma}{(4\pi)^3 \cdot R_{RT}^4} \\
C = \frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma_c}{(4\pi)^3 \cdot R_{RT}^4} \\
\frac{S}{C} = \frac{\frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma}{(4\pi)^3 \cdot R_{RT}^4}}{\frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma_c}{(4\pi)^3 \cdot R_{RT}^4}} = \frac{\sigma}{\sigma_c}
\]

The clutter RCS and clutter area are defined by Equations 6-25 and 6-26, respectively. We incorporate these equations into the S/C equation. The resultant equation below is the same as Equation 7-1.

\[
\sigma_c = \sigma_0 \cdot A_c \\
A_c = \left( R \cdot \theta_{3dB} \right) \cdot \left( \frac{c \cdot \tau}{2} \right) \cdot \left( \frac{1}{\cos(\alpha)} \right)
\]

\[
\frac{S}{C} = \frac{\sigma}{\sigma_0 \cdot \left[ \left( R \cdot \theta_{3dB} \right) \cdot \left( \frac{c \cdot \tau}{2} \right) \cdot \left( \frac{1}{\cos(\alpha)} \right) \right]} = \frac{2 \cos(\alpha) \cdot \sigma}{\sigma_0 \cdot R \cdot \theta_{3dB} \cdot c \cdot \tau}
\]
**Exercise 7-4** A focused side-looking SAR system, traveling at velocity $V_R$ with plenty of sensitivity has a $\lambda/D$ radian beamwidth.

(a) Calculate the Doppler spread across the beam, $f_{d_{max}}$, at range $R$ and at range $2R$.

(b) What is time it takes for the beam to transit a target location, $T_I$, at those two ranges?

(c) How many cross range resolution cells (Doppler filters) at $R$ and at $2R$?

(d) What is the cross range resolution, $d_a$, at $R$, at $2R$?

(a) The maximum Doppler spread, $f_{d_{max}}$ (Hz), is given by Equation 7-32.

$$f_{d_{max}} = \frac{2 \cdot V \cdot \theta_{3dB}}{\lambda}$$

We use the terms given in the exercise and obtain the maximum Doppler spread, as shown below. Because $f_{d_{max}}$ is independent of range, this value is the same for both ranges.

$$f_{d_{max}} = \frac{2 \cdot V \cdot \left(\frac{\lambda}{D}\right)}{\lambda} = \frac{2 \cdot V}{D}$$

(b) Equation 7-27 defines the time it takes the target to transit from the left edge to the right edge of the beam or time the target is illuminated by the beam, $T_I$ (seconds).

$$T_I = \frac{R \cdot \theta_{3dB}}{V_{ac}}$$

We use the terms given in the exercise to compute the time for both ranges, as shown below:

$$T_I = \frac{R \cdot \left(\frac{\lambda}{D}\right)}{V_{ac}} = \frac{R \cdot \lambda}{V_{ac} \cdot D} \quad T_I = \frac{2R \cdot \left(\frac{\lambda}{D}\right)}{V_{ac}} = \frac{2R \cdot \lambda}{V_{ac} \cdot D}$$
(c) The number of Doppler filters, \( N_f \) (no units), is given by Equation 7-33.

\[
N_f = \frac{2 \cdot R \cdot \theta_{3dB}}{\lambda}^2
\]

We use the terms given in the exercise to compute the number of Doppler filters for both ranges, as shown below:

\[
N_f = \frac{2 \cdot R \cdot \left( \frac{\lambda}{D} \right)^2}{\lambda} = \frac{2 \cdot R \cdot \lambda}{D^2}
\]

\[
N_f = \frac{2 \cdot (2 \cdot R) \cdot \left( \frac{\lambda}{D} \right)^2}{\lambda} = \frac{4 \cdot R \cdot \lambda}{D^2}
\]

(d) The cross range resolution, \( d_a \) (m), is given by Equation 7-34.

\[
d_a = \frac{D}{2}
\]

Because \( d_a \) is independent of range, this value is the same for both ranges.
Exercise 7-5  A focused side-looking SAR with a wavelength \( \lambda = 0.03 \) m is on an aircraft traveling at \( V_R = 300 \) m/sec.  (a) What is the Doppler filter bandwidth needed to resolve two points separated by \( \Delta \theta = 0.005 \) degrees, \( \Delta f_d \) (Hertz)?  (b) What is the required integration time, \( T_I \) (seconds)?  (c) What is the synthetic array length, \( L \) (meters)?  (d) What is the cross range resolution at a range \( R_p = 50 \) km, \( d_a \) (meters)?  Repeat parts (c) and (d) if the radar antenna mainbeam is squinted at \( \theta_p = 45 \) degrees and the elevation angle to the patch on the ground is \( \phi_p = 10 \) degrees.

We start by defining the values for all the terms and necessary constants.

\[
\lambda \equiv 0.03 \text{ m} \quad V_R \equiv 300 \cdot \frac{\text{m}}{\text{sec}} \quad \Delta \theta \equiv 0.005 \cdot \text{deg} \quad R_p \equiv 50 \cdot 10^3 \cdot \text{m} \quad \theta_p \equiv 45 \cdot \text{deg} \quad \phi_p \equiv 10 \cdot \text{deg}
\]

(a) The Doppler filter bandwidth, \( \Delta f_d \) (Hertz), needed to resolve two points separated in angle is given in Equation 7-23.  Note: this equation has \( \Delta \theta \) in radians and thus we need to convert the provided value from degrees to radians (\( \pi / 180 \) deg).

\[
\Delta f_d := \frac{2 \cdot V_R \cdot \left( \Delta \theta \times \frac{\pi}{180 \cdot \text{deg}} \right)}{\lambda} = 1.7453 \cdot \text{Hz}
\]

(b) The integration time, \( T_I \) (seconds), for a focused side-looking SAR is given in Equation 7-25.

\[
\Delta f_d = \frac{1}{T_I} \quad T_I := \frac{1}{\Delta f_d} = 0.573 \text{s} \quad \frac{1}{(1.7453 \cdot \text{Hz})} = 0.573 \text{s}
\]

(c) The synthetic array length, \( L \) (meters), for a focused side-looking SAR is given in Equation 7-28.

\[
L := V_R \cdot T_I = 171.8873 \text{ m} \quad \text{m} \cdot \text{sec} \cdot (0.573 \cdot \text{s}) = 171.9 \text{ m}
\]
(d) The cross range resolution, $d_a$ (meters), for a focused side-looking SAR is given in Equation 7-31.

$$d_a := \frac{R_p \lambda}{2 \cdot L} = 4.3633 \text{ m}$$

For a squinted SAR the synthetic array length, $L_s$ (meters), and cross range resolution, $d_a$ (meters), include the azimuth, $\theta_p$ (degrees), and elevation, $\phi_p$ (degrees), angles relative to the velocity vector to the patch on the ground. Equation 7-38 includes these angles. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with $\theta_p$ and $\phi_p$ indicates these angles are in degrees (like the "DEG" button on your calculator).

$$L_s := V_R \cdot \sin(\theta_p) \cdot \cos(\phi_p) \cdot T_I = 119.6962 \text{ m}$$

$$R_p \cdot \frac{\lambda}{2 \cdot L_s} = 6.2659 \text{ m}$$

$$R_p \cdot \frac{\lambda \cdot \Delta f_d}{2 \cdot V_R \cdot \sin(\theta_p) \cdot \cos(\phi_p)} = 6.2659 \text{ m}$$

$$R_p \cdot \frac{\lambda}{2 \cdot V_R \cdot \sin(\theta_p) \cdot \cos(\phi_p) \cdot T_I} = 6.2659 \text{ m}$$
**Exercise 7-6** A focused side-looking SAR system maps a 16 km-wide by 16 km long swath in $T_I = 2$ minutes. With a pulse bandwidth $B_{pc} = 100$ MHz and a real beamwidth of $\theta_{3dB} = 2$ degrees at $f_c = 3000$ MHz, what would be the number of resolution cells processed per second?

We start by defining the values for all the terms and necessary constant.

$$R_{wide} \equiv 16 \times 10^3 \cdot m \quad R_{long} \equiv 16 \times 10^3 \cdot m \quad T_I \equiv 2 \times 60 \cdot \text{sec} = 120 \text{ s} \quad B_{pc} \equiv 100 \times 10^6 \cdot \text{Hz} \quad \theta_{3dB} \equiv 2 \cdot \text{deg}$$

$$f_c \equiv 3000 \times 10^6 \cdot \text{Hz} \quad c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}$$

The number of range resolution cells is map swath length divided by the range resolution. We were given the range swath length. The range resolution can be computed from the waveform bandwidth using Equation 5-16.

$$\Delta R := \frac{c}{2 \cdot B_{pc}} = 1.5 \text{ m}$$

$$N_{\text{range}} := \frac{R_{long}}{\Delta R} = 1.0667 \times 10^4$$

The number of cross range resolution cells is equal to the number of Doppler filters, as given in Equation 7-33. Note: in this equation $\theta_{3dB}$ is in radians, thus we convert it from degrees to radians ($\pi / 180$ deg). We need to compute the wavelength, Equation 2-2, Doppler filter bandwidth, Equation 7-25, and aircraft speed. The aircraft speed is the swath width divided by the time it takes the target to transit for the left edge of the beam to the right.

$$\lambda := \frac{c}{f_c} = 0.1 \text{ m}$$

$$N_f = \frac{2 \cdot V_{ac} \cdot \theta_{3dB}}{\lambda \cdot \Delta f_d}$$
\[ \Delta f_d := \frac{1}{T_I} = 8.333 \times 10^{-3} \, \text{Hz} = \frac{1}{(120 \cdot s)} = 8.333 \times 10^{-3} \, \text{Hz} \]

\[ V_{ac} := \frac{R_{\text{wide}}}{T_I} = 133.333 \frac{m}{s} = \frac{16 \cdot 10^3 \cdot m}{(120 \cdot s)} = 133.333 \frac{m}{s} \]

\[ 2 \cdot V_{ac} \left( \theta_{\text{3dB}} \cdot \frac{\pi}{180 \cdot \text{deg}} \right) \]

\[ N_f := \frac{\lambda}{\Delta f_d} = 1.117 \times 10^4 \]

\[ \frac{2 \left[ 133.333 \frac{m}{s} \right] \cdot 2 \cdot \text{deg} \cdot \frac{\pi}{(180 \cdot \text{deg})}}{\left[ 3.10^8 \frac{m}{(3000 \cdot 10^6 \cdot \text{Hz})} \right]} = \frac{1.117 \times 10^4}{(8.3333 \times 10^{-3} \cdot \text{Hz})} = 1.117 \times 10^4 \]

The total number of resolution cells, \( \text{Total cells} \) (no units), is the product of the number of range cells and the number of Doppler filters. The number of resolution cells processed per second, \( \text{Cells per sec} \) (cells/second), is the total number of resolution cells divided by the time it takes to collect them. Note: the units on \( \text{Cells per sec} \) is "one over seconds" or "per second".

\[ \text{Total cells} := N_{\text{range}} \cdot N_f = 1.1915 \times 10^8 \]

\[ = 10666.666666666666 \cdot 11170.107212763711 = 1.1915 \times 10^8 \]

\[ \text{Cells per sec} := \frac{\text{Total cells}}{T_I} = 9.929 \times 10^5 \frac{1}{s} \]

\[ = \frac{11947810.26947957}{(120 \cdot s)} = 9.929 \times 10^5 \frac{1}{s} \]
**Exercise 7-7**  A side-looking focused SAR with an antenna \( D = 1 \) meter in diameter is imaging a range extent from 10 km to 100 km.  (a) What is the maximum PRF required to avoid range ambiguities, \( \text{PRF}_{\text{max}} \) (Hertz)?  (b) At this PRF, what is the maximum platform speed to avoid Doppler ambiguities (assuming null-to-null beamwidth), \( V_R \) (m/sec)?  Repeat part (b) if the radar antenna mainbeam is squinted at \( \theta_p = 60 \) degrees and the elevation angle to the patch on the ground is \( \phi_p = 6 \) degrees.

We start by defining the values for all the terms and the necessary constant.

\[
\begin{align*}
D & \equiv 1 \text{ m} \\
R_{\text{near}} & \equiv 10 \times 10^3 \text{ m} \\
R_{\text{far}} & \equiv 100 \times 10^3 \text{ m} \\
\theta_p & \equiv 60 \text{ deg} \\
\phi_p & \equiv 5 \text{ deg} \\
c & \equiv 3 \times 10^8 \text{ m/sec}
\end{align*}
\]

(a) The maximum PRF required to avoid range ambiguities, \( \text{PRF}_{\text{max}} \) (Hertz), for a focused side-looking SAR is given in Equation 7-35.

\[
\text{PRF}_{\text{max}} = \frac{c}{2R_{\text{sw}}}
\]

\[
\begin{align*}
\text{PRF}_{\text{max}} & := \frac{c}{2(R_{\text{far}} - R_{\text{near}})} = 1.6667 \times 10^3 \text{ Hz} \\
& = \frac{3 \times 10^8 \left( \frac{\text{m}}{\text{sec}} \right)}{2 \left( 100 \times 10^3 \text{ m} - 10 \times 10^3 \text{ m} \right)} = 1.6667 \times 10^3 \text{ Hz}
\end{align*}
\]

(b) The minimum PRF required to avoid range ambiguities, \( \text{PRF}_{\text{min}} \) (Hertz), for a focused side-looking SAR is a function of the platform speed, as given in Equation 7-36.  We solve this equation for the maximum platform speed, \( V_R \) (meters/second), to avoid Doppler ambiguities when using the PRF from part (a).

\[
\text{PRF}_{\text{min}} = \frac{4V_R}{D}
\]

\[
\begin{align*}
\text{PRF}_{\text{min}} & := \text{PRF}_{\text{max}} = 1.6667 \times 10^3 \text{ Hz} \\
V_R & := \frac{\text{PRF}_{\text{min}} \cdot D}{4} = 416.6667 \text{ m/s}
\end{align*}
\]

\[
V_R = \frac{1.6667 \times 10^3 \text{ Hz} \cdot (1 \text{ m})}{4} = 416.675 \text{ m/s}
\]
For a squinted SAR the Doppler-related equations include the azimuth, $\theta_p$ (degrees), and elevation, $\phi_p$ (degrees), angles relative to the velocity vector to the patch on the ground. Equation 7-38 shows how these angles are included with the platform speed. Thus, we can include the angles in the same manner in the minimum PRF equation. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with $\theta_p$ and $\phi_p$ indicates these angles are in degrees (like the "DEG" button on your calculator).

$$\text{PRF}_{\text{min}} = \frac{4 \cdot V_R \cdot \sin(\theta_p) \cdot \cos(\phi_p)}{D}$$

$$V_R := \frac{\text{PRF}_{\text{max}} \cdot D}{4 \cdot \sin(\theta_p) \cdot \cos(\phi_p)} = \frac{1.6667 \cdot 10^3 \text{ Hz} \cdot (1 \text{ m})}{(4 \cdot \sin(60 \text{ deg}) \cdot \cos(5 \text{ deg}))} = \frac{482.9727 \text{ m}}{\text{s}}$$
**Exercise 7-8** A spotlight SAR operates at a carrier frequency $f_c = 5.4 \text{ GHz}$ on a platform with a speed of $V_R = 400 \text{ m/sec}$.  (a) What is the finest possible cross range resolution for $T_I = 10 \text{ sec}$ of spotlight time at a range $R_p = 185 \text{ km}$, $d_a$ (meters)?  (b) What is the required minimum PRF to avoid Doppler ambiguities for a radar antenna null-to-null beamwidth $\theta_{nn} = 1.5 \text{ degrees}$, $PRF_{\text{min}}$ (Hertz)?  (c) What is the maximum slant range extent of the image, swath width, at the minimum PRF before range ambiguities occur, $R_{sw}$ (meters)?

We start by defining the values for all the terms and necessary constants.

$$f_c \equiv 5.4 \times 10^9 \text{ Hz} \quad V_R \equiv 400 \cdot \frac{\text{m}}{\text{sec}} \quad T_I \equiv 10 \cdot \text{sec} \quad R_p \equiv 185 \times 10^3 \cdot \text{m} \quad \theta_{nn} \equiv 1.5 \cdot \text{deg}$$

$$c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}}$$

(a) The finest possible cross range resolution, $d_a$ (meters), for a spotlight SAR is given in Equation 7-31. We need the wavelength, $\lambda$ (meters), and will compute it using Equation 2-2.

$$\lambda := \frac{c}{f_c} = 0.0556 \text{ m}$$

$$d_a := R_p \frac{\lambda}{2 \cdot V_R \cdot T_I} = 1.2847 \text{ m}$$

(b) The minimum PRF to avoid Doppler ambiguities, $PRF_{\text{min}}$ (Hertz), is given in 7-36. Note: This equation needs the null-to-null beamwidth in radians. Thus, the conversion of $\theta_{nn}$ from degrees to radians ($\pi / 180 \text{ deg}$).
PRF_{\text{min}} := \frac{2 \cdot V_R \cdot \left( \theta_{\text{nn}} \times \frac{\pi}{180 \cdot \text{deg}} \right)}{\lambda} = 376.9911 \cdot \text{Hz}

\[
\begin{align*}
&\quad = \frac{2 \cdot 400 \cdot \left( \frac{\text{m}}{\text{sec}} \right) \cdot 1.5 \cdot \text{deg} \cdot \frac{\pi}{(180 \cdot \text{deg})}}{3 \cdot 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right) \left( 5.4 \cdot 10^9 \cdot \text{Hz} \right)} \\
&\quad = \frac{3 \cdot 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right)}{2 \cdot (376.9911 \cdot \text{Hz})} \\
&\quad = 3.9789 \times 10^5 \text{m}
\end{align*}
\]

(c) The maximum PRF to avoid range ambiguities is a function of the range swath width, R_{sw} (meters), as given in Equation 7-35. We will solve this equation for the range swath width provided by the minimum PRF obtained in part (b).

\[\text{PRF}_{\text{max}} = \frac{c}{2 \cdot R_{sw}}\]

\[R_{sw} := \frac{c}{2 \cdot \text{PRF}_{\text{min}}} = 3.9789 \times 10^5 \text{m}\]
**Exercise 7-9** A radar system with a Doppler Beam Sharpening (DBS) mode has the following characteristics: carrier frequency \( f_c = 10 \ \text{GHz} \) and Doppler filter bandwidth \( \Delta f_d = 15 \ \text{Hz} \). The radar speed is \( V_R = 280 \ \text{m/sec} \). (a) What is the DBS cross range resolution, \( \text{DBS}_a \) (meters), for the following geometries to the patch on the ground: range \( R_p = 15 \ \text{km} \), elevation angle \( \phi_p = 42 \ \text{degrees} \), at two azimuth angles \( \theta_p = 60 \ \text{degrees} \) and \( 30 \ \text{degrees} \)? (b) Repeat part (a) for the following geometry to the patch on the ground: range \( R_p = 30 \ \text{km} \) and elevation angle \( \phi_p = 30 \ \text{degrees} \)? Compare your results to those in Figure 7-14.

We start by defining the values for all the terms and the necessary constants.

\[
\begin{align*}
f_c & \equiv 10 \times 10^9 \ \text{Hz} \\
\Delta f_d & \equiv 15 \ \text{Hz} \\
V_R & \equiv 280 \cdot \frac{\text{m}}{\text{sec}} \\
R_p1 & \equiv 15 \times 10^3 \ \text{m} \\
\phi_p1 & \equiv 42 \ \text{deg} \\
\theta_p1 & \equiv 60 \ \text{deg} \\
\theta_p2 & \equiv 30 \ \text{deg} \\
R_p2 & \equiv 30 \times 10^3 \ \text{m} \\
\phi_p2 & \equiv 20 \ \text{deg} \\
c & \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}} \\
\end{align*}
\]

(a) The Doppler beam sharpening (DBS) cross range resolution, \( \text{DBS}_a \) (meters), is given in Equation 7-39 and limited (because a DBS is unfocused) by Equation 7-40. These equations need the wavelength, \( \lambda \) (meters), so we will compute it using Equation 2-2. Note: Mathcad uses radians as the base units for angles, so we need to be careful with degrees and radians. Thus, the "deg" with \( \theta_p \) and \( \phi_p \) indicates these angle are in degrees (like the "DEG" button on your calculator).

\[
\lambda := \frac{c}{f_c} = 0.03 \ \text{m}
\]

\[
\frac{\lambda \cdot \Delta f_d \cdot R_p1}{2 \cdot V_R \cdot \sin(\theta_p1) \cdot \cos(\phi_p1)} = 18.7289 \ \text{m}
\]
As shown above, for this condition \((R_{p1}, \phi_{p1}, \theta_{p1})\) DBS\(_a\) is limited to 23.3 meters. This result is the same as in Figure 7-14 (which is also shown at the end of this solution sheet).

The calculations for the next condition are:

\[
\frac{\lambda \Delta f_d R_{p1}}{2 V_R \sin(\theta_{p2}) \cos(\phi_{p1})} = 32.4394 \text{m}
\]
\[
\sqrt{\frac{\lambda \cdot R_{p1}}{2 \cdot \sin(\theta_{p2}) \cdot \cos(\phi_{p1})}} = 40.369 \text{m}
\]

As shown above, for this condition \((R_{p1}, \phi_{p1}, \text{and} \theta_{p2})\) \(\text{DBS}_a\) is limited to 40.4 meters. This result is the same as in Figure 7-14.

(b) The calculations for the next condition are:

\[
\sqrt{\frac{\lambda \cdot \Delta f_d \cdot R_{p2}}{2 \cdot V_R \cdot \sin(\theta_{p1}) \cdot \cos(\phi_{p2})}} = 29.623 \text{m}
\]

\[
= \sqrt{\frac{3 \cdot 10^8 \cdot \left(\frac{\text{m}}{\text{sec}}\right)}{(10 \cdot 10^9 \cdot \text{Hz})} \cdot (15 \cdot 10^3 \cdot \text{m}) \cdot \frac{1}{2 \cdot \left[280 \cdot \left(\frac{\text{m}}{\text{sec}}\right) \cdot \sin(60\cdot \text{deg}) \cdot \cos(20\cdot \text{deg})\right]}} = 29.623 \text{m}
\]

\[
\sqrt{\frac{\lambda \cdot R_{p2}}{2 \cdot \sin(\theta_{p1}) \cdot \cos(\phi_{p2})}} = 26.0669 \text{m}
\]

\[
= \sqrt{\frac{3 \cdot 10^8 \cdot \left(\frac{\text{m}}{\text{sec}}\right)}{(10 \cdot 10^9 \cdot \text{Hz})} \cdot (30 \cdot 10^3 \cdot \text{m}) \cdot \frac{1}{2 \cdot \left[\sin(60\cdot \text{deg}) \cdot \cos(20\cdot \text{deg})\right]}} = 26.0669 \text{m}
\]
As shown above, for this condition \((R_{p2}, \phi_{p2}, \text{ and } \theta_{p1})\) DBS\(_a\) is 29.6 meters. This result is the same as in Figure 7-14.

The calculations for the next condition are as follows:

\[
\frac{\lambda \cdot \Delta f_d \cdot R_{p2}}{2 \cdot V_R \cdot \sin(\theta_{p2}) \cdot \cos(\phi_{p2})} = 51.3086 \text{ m}
\]

\[
\sqrt{\frac{\lambda \cdot R_{p2}}{2} \cdot \frac{1}{\sin(\theta_{p2}) \cdot \cos(\phi_{p2})}} = 45.1492 \text{ m}
\]

As shown above, for this condition \((R_{p2}, \phi_{p2}, \text{ and } \theta_{p2})\) DBS\(_a\) is 51.3 meters.

The equation and plot used for Figure 7-14 are shown below.

\[
D_a(R_p, \phi_p, \theta_p) := \begin{aligned}
\text{temp} & \leftarrow \frac{\lambda \cdot \Delta f_d \cdot R_p}{2 \cdot V_R \cdot \sin(\theta_p \cdot \text{deg}) \cdot \cos(\phi_p)} \\
\text{limit} & \leftarrow \sqrt{\frac{\lambda \cdot R_p}{2} \cdot \frac{1}{\sin(\theta_p \cdot \text{deg}) \cdot \cos(\phi_p)}} \\
\text{max} & \leftarrow (\text{temp, limit})
\end{aligned}
\]
Exercise 7-10  A bistatic radar system uses an FM radio station for a transmitter: peak power $P_R = 50$ kW, transmit antenna gain in the direction of the target $G_{RT} = 0$ dBi, and frequency $f_c = 100$ MHz. The receiver has the following characteristics: receive antenna gain in the direction of the target $G_{RT} = 10$ dBi, signal processing gain $G_{SP} = 15$, noise figure $F_R = 6$ dB, bandwidth $B_R = 200$ kHz, and radar system losses $L_R = 7$ dB. If the detection threshold $SNR_{dt} = 13$ dB, what is the bistatic range product for a target bistatic radar cross section $\sigma_b = 5$ m$^2$?

We start by defining the values for all the terms and necessary constants. Convert values from decibels to absolute as needed.

\[
\begin{align*}
PR & \equiv 50 \times 10^3 \text{ W} & G_{RT \text{ dBi}} & \equiv 0 \\
SNR_{dt \text{ dB}} & \equiv 13 & F_R \text{ dB} & \equiv 6 \\
k & \equiv 1.38 \times 10^{-23} \frac{\text{ W \cdot sec}}{\text{ K}} \\
G_{RT} & := 10 \left( \frac{G_{RT \text{ dBi}}}{10} \right) = 1 \\
L_R & := 10 \left( \frac{L_R \text{ dB}}{10} \right) = 5.0119 \\
\lambda & \equiv \frac{c}{f_c} = 3 \times 10^8 \cdot \frac{\text{ m}}{\text{ sec}} \\
F_R & := 10 \left( \frac{F_R \text{ dB}}{10} \right) = 3.9811 \\
SNR_{dt} & := 10 \left( \frac{SNR_{dt \text{ dB}}}{10} \right) = 19.9526 \\
\end{align*}
\]

The bistatic detection range product, $R_1R_2$ (m$^2$), is computed using Equation 7-42. We were given values for almost all the terms in this equation. All we need to do is compute the wavelength, $\lambda$ (m), using Equation 2-2.
\[ \lambda := \frac{c}{f_c} = 3 \text{m} \]

\[ = \frac{3 \times 10^8 \frac{\text{m}}{\text{sec}}}{(100 \times 10^6 \cdot \text{Hz})} = 3 \text{m} \]

\[ R1R2 := \sqrt{\frac{P_R \cdot G_{RT} \cdot G_{RT} \cdot \lambda^2 \cdot \sigma_b \cdot G_{SP}}{(4\pi)^3 \cdot SNR_{dt} \cdot F_R \cdot k \cdot T_0 \cdot B_R \cdot L_R}} = 7.3058 \times 10^8 \text{m}^2 \]

\[ R1R2 = 730.5813 \text{km}^2 \]

\[ (50 \times 10^3 \cdot \text{W}) \left[ \frac{0}{10} \right] \left[ \frac{10}{10} \right] \left[ \frac{10^8}{100 \times 10^6} \frac{\text{m}}{\text{Hz}} \right] \left[ \frac{5}{2} \right] = 7.3058 \times 10^8 \text{m}^2 \]
**Exercise 7-11** A multifunction radar system uses a search waveform providing an unambiguous range $R_u = 150 \text{ km}$ and integrates $n_p = 32$ pulses. The resource manager needs to develop a waveform to update an existing track of a target at range $R_{RT} = 60 \text{ km}$. (a) If the search waveform is used to provide the track update what is the integration time, $T_I$ (seconds)? (b) A customized update waveform can be developed based on the estimated target range. Since the target range is only an estimate the customized update waveform is based on providing an unambiguous range 5% greater than the target range. What is the pulse repetition frequency, $PRF$ (Hertz), and integration time, $T_I$ (seconds), for the customized update waveform? (c) How much time was saved by using the customized update waveform and how many additional search waveforms (beam positions) could be transmitted in that time?

We start by defining the values for all the terms and necessary constants.

$$R_u = 150 \times 10^3 \text{ m} \quad n_p = 32 \quad R_{RT} = 60 \times 10^3 \text{ m} \quad c = 3 \times 10^8 \text{ m/sec}$$

(a) We first determine the pulse repetition frequency, $PRF$ (Hertz), necessary to provide the unambiguous range, as given in Equation 5-20. We know the number of pulses integrated, $n_p$ (no units), is a function of the integration time and $PRF$, as given in Equation 3-41. We solve this equation for the integration time, $T_I$ (seconds). These equations and calculations are shown below.

$$PRF_1 := \frac{c}{2 \cdot R_u} = 1 \times 10^3 \text{ Hz}$$

$$n_p = T_I \cdot PRF$$

$$T_I := \frac{n_p}{PRF_1} = 0.032 \text{ s}$$

(b) We first determine the unambiguous range that is 5% greater than the target range. We then compute the $PRF$ and integration time associated with this unambiguous range, using the equations from part (a). These equations and calculations are shown below:

$$R_{u2} := (1 + 0.05) \cdot R_{RT} = 6.3 \times 10^4 \text{ m}$$

$$= (1 + 0.05) \cdot (60 \times 10^3 \text{ m}) = 6.3 \times 10^4 \text{ m}$$

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PRF$_2$ := \( \frac{c}{2 \cdot R_{u2}} = 2.381 \times 10^3 \cdot Hz \)

\[
\begin{aligned}
PRF_2 &= \frac{c}{2 \cdot R_{u2}} \\
&= \frac{3 \cdot 10^8 \cdot \left( \frac{m}{sec} \right)}{2 \cdot \left( 6.3 \times 10^4 \cdot m \right)} \\
&= 2.381 \times 10^3 \cdot Hz
\end{aligned}
\]

\( T_{I2} := \frac{n_p}{PRF_2} = 0.0134 \text{ s} \)

\[
\begin{aligned}
T_{I2} &= \frac{n_p}{PRF_2} \\
&= \frac{32}{2.381 \cdot 10^3 \cdot Hz} = 0.0134 \text{ s}
\end{aligned}
\]

(c) The time saved by using the customized update waveform is the difference between the integration times from parts (a) and (b).

\[
T_{I1} - T_{I2} = 0.0186 \text{ s}
\]

The number of additional search waveforms (beam positions) that could be transmitted is the ratio of the two integration times from parts (a) and (b).

\[
\begin{aligned}
\frac{T_{I2}}{T_{I1}} &= 0.42 \\
&= \frac{0.0134 \cdot s}{(0.032 \cdot s)} = 0.4188
\end{aligned}
\]

Note: this is the correct answer not the one in the back of the book

While this value alone is not significant, if the radar system had numerous established target tracks the time saved from all the target updates can become significant.

This exercise represents just the "tip of the iceberg" about waveform development based on target track information. The options are bounded only by a radar designer's imagination, radar theory, detection theory, and probability theory.
**Exercise 7-12** An over-the-horizon backscatter radar has its receiver facility located 40 km from the transmitter facility and at right angles to the transmit antenna's boresight. The transmit and receive antenna beams scan ±30 degrees simultaneously. Both are unweighted apertures with 26 dBi mainbeam gain at 30 MHz.  

(a) At this frequency, what is the approximate isolation afforded by the geographic separation of the two facilities? (Hint: Consider a flat earth. Assume the sidelobe antenna gain is one-one hundredth (-20 dB) the mainbeam antenna gain.)

(b) The system employs an unweighted FM/CW waveform using an FM ramp of 10 millisecond duration with a bandwidth of $B_{pc} = 1$ MHz. The minimum range is 740 km. What additional isolation is afforded by this waveform? (Hint: Assume waveform time sidelobes are $[\sin(x)/x]^2$.)

(c) If the effective radiated power of the OTH-B system were 80 dBW, what signal level direct from the transmitter would enter the receiver front end? 

(d) Would appear in the first range bin (740 km) at the output of the FM/CW processor? (e) A 10 m² target at 740 km range would enter the receiver at -133 dBW, assuming $1/R^4$ attenuation, and would appear at the processor output at about 10 dB below the transmitter power. What are your comments about the need for additional suppression of the direct transmitter signal?

We start by defining the values for all the terms and necessary constants.

$$f_c = 30 \times 10^6 \text{ Hz}$$

$$R_{TR} = 40 \times 10^3 \text{ m}$$

$$G_{\text{dBi}} = 26$$

$$R_{\text{min}} = 740 \times 10^3 \text{ m}$$

$$\tau = 10 \times 10^{-3} \text{ sec}$$

$$B_{pc} = 1 \times 10^6 \text{ Hz}$$

$$R = 740 \times 10^3 \text{ m}$$

$$ERP_{\text{dBW}} = 80$$

$$\sigma = 10 \text{ m}^2$$

$$c = 3 \times 10^8 \frac{\text{ m}}{\text{ sec}}$$

Convert values from dB into absolute units and compute the wavelength, $\lambda$ (m), using Equation 2-2.

\[
G := 10 \left( \frac{26}{10} \right) = 398.1072
\]

\[
ERP := 10 \left( \frac{80}{10} \right) \cdot W = 1 \times 10^8 W
\]

\[
\lambda := \frac{c}{f_c} = 10\text{ m}
\]

(a) The approximate isolation afforded by the geographic separation of the two facilities (transmitter and receiver) is obtained by deriving the the transmitter-to-receiver equation. We do not have this equation in the discussion on OTH radars. But we can use what is presented in Chapter 2 and the support jammer discussion in Chapter 11, and build from there. The power at the OTH receiver that comes directly from the OTH transmitter, $P_{\text{direct}}$ (Watts), is derived in a similar manner as the support jammer power equation, Equation 11-5. We start with the OTH transmitter
power density that hits the OTH receive antenna and multiple it by the area of the OTH receive antenna. We then use the relationship between antenna area and gain, Equation 2-8. We simplify this equation and are left with $P_{\text{direct}}$. All the equations are shown below:

$$P_{\text{direct}} = \frac{P \cdot G_T}{4\pi R_{RT}^2} \cdot A_R = \frac{P \cdot G_T \cdot G_R \cdot \lambda^2}{(4\pi)^2 \cdot R_{RT}^2} \quad A_R = \frac{G_R \cdot \lambda^2}{4\pi}$$

The signal from the OTH transmitter enters the receive antenna through a sidelobe. The level of the sidelobe is $G/100$. Because the transmit and receive antennas are identical we can replace $G_T$ and $G_R$ with the same antenna gain term, $G$. We will continue to keep the transmit antenna gain and receive antenna gain separate to help identify the isolation we are attempting to compute.

$$P_{\text{direct}} = \frac{P \cdot G \left(\frac{G}{100}\right) \cdot \lambda^2}{(4\pi)^2 \cdot R_{TR}^2}$$

Removing the transmit power and antenna gain gives the isolation afforded by the separation of the transmitter and receiver, Isolation (no units). It does not matter what the transmit power and antenna gain are, the transmitter-receiver separation and receive antenna gain in the direction of the transmitter ($G/100$ in this exercise), is the isolation.

$$\text{Isolation} := \frac{G}{100} \cdot \lambda^2 \cdot \frac{1}{(4\pi)^2 \cdot R_{TR}^2} = 1.5757 \times 10^{-9}$$

$$= \frac{398.10717055349733}{100} \cdot \left[\frac{3 \cdot 10^8 \cdot \left(\frac{\text{m}}{\text{sec}}\right)}{30 \cdot 10^6 \cdot \text{Hz}}\right]^2 = 1.5757 \times 10^{-9}$$

$$\text{Isolation}_{\text{dB}} := 10 \cdot \log (\text{Isolation}) = -88.0254$$
(b) The additional isolation afforded by the FM/CW waveform can be determined using the discussion of time/range sidelobes in Section 5.5.4. As shown in Figure 5-46, the time sidelobes repeat themselves in \( \Delta R \) (range resolution) bins in the range domain. We need to determine the number of \( \Delta R \) bins, \( n_{rg} \) (no units), between the minimum range of the radar, \( R_{min} \) (m), and the transmitter-receiver range, \( R_{TR} \) (m). We compute the range resolution for a radar with pulse compression using Equation 5-16. We can then compute isolation provided by the level of the time sidelobe associated with the transmitter-to-receiver range, \( \text{Isolation\_FMCW} \) (no units and dB). All the values and equations are shown below:

\[
\Delta R := \frac{c}{2 \cdot B_{pc}} = 150 \text{m}
\]

\[
n_{rg} := \frac{R_{min} - R_{TR}}{\Delta R} = 4.6667 \times 10^3
\]

\[
\text{Isolation\_FMCW} := \left( \frac{\sin(n_{rg})}{n_{rg}} \right)^2 = 4.4616 \times 10^{-8}
\]

\[
\text{Isolation\_FMCW\_dB} := 10 \cdot \log(\text{Isolation\_FMCW}) = -73.5051
\]

(c) The signal level direct from the transmitter that would enter the receiver front end is given by the following equation.

Note: The "1W" in the denominator of the decibels equation is for Watts, and is needed to compute decibels relative to a Watt (dBW).

\[
P_{direct} := \frac{\text{ERP} \cdot \left( \frac{G}{100} \right) \cdot \lambda^2}{(4\pi)^2 \cdot R_{TR}^2} = 0.1576 \text{W}
\]
$\frac{1 \cdot 10^8 \cdot \text{W} \cdot \left(\frac{398.10717055349733}{100}\right)}{\left[\frac{3 \cdot 10^8 \cdot \left(\frac{\text{m}}{\text{sec}}\right)}{30 \cdot 10^6 \cdot \text{Hz}}\right]^2} = 0.1576 \text{W}$

\[ P_{\text{direct dbW}} := 10 \cdot \log \left(\frac{P_{\text{direct}}}{1 \text{W}}\right) = -8.0254 \]

\[ = 10 \cdot \log \left(\frac{0.1576 \cdot \text{W}}{(1 \cdot \text{W})}\right) = -8.0244 \]

(d) The signal level at the output of the FM/CW processor, Out (W), is the product of the isolation afforded by the FM/CW waveform, part (b), and the signal level direct from the transmitter, part c). We can sum the isolation and the signal direct from the transmitter in decibels, Out_dB (dBW), as shown below:

\[ \text{Out}_\text{dB} := \text{Isolation}_{\text{FMCW}} \text{ dB} + P_{\text{direct dbW}} = -81.5305 \]

\[ = -73.505096073429144 - 8.0253971070011723 = -81.5305 \]

(e) The target signal would enter the receiver at -133 dBW and would appear at the output of the processor at a value of \(\Delta\_\text{dBW}\) (dBW) relative to the transmitter signal. The values and equations are shown below:

\[ \Delta\_\text{dBW} := -133 - \text{Out}_\text{dB} = -51.4695 \]

\[ = -133 - -81.530493180430312 = -51.4695 \]

Because \(\Delta\_\text{dBW}\) is much less than 0 dB, additional suppression is needed for a target with this RCS and range from the OTH radar.
**Exercise 7-13** What is the projected diameter of an optical reflector (cat's eye) with a laser radar cross section of 1 million square meters? (Assume the reflectivity $\rho = 1$ and use a wavelength of 0.5 micrometers.)

We start by defining the values for all the terms.

$$\sigma \equiv 1 \times 10^6 \text{ m}^2 \quad \rho \equiv 1 \quad \lambda \equiv 0.5 \times 10^{-6} \text{ m}$$

In the text on Lidar RCS we see the RCS of a target is given in Equation 7-54. We can solve this equation for the effective intercept area, $A$ (m$^2$), and use it to compute the diameter, $D$ (m), of the circle which provides this area. All the equations and numbers are shown below.

$$\sigma = \rho \cdot \frac{4\pi \cdot A^2}{\lambda^2}$$

$$A := \sqrt[\lambda^2]{\frac{\sigma \cdot \lambda^2}{4\pi \cdot \rho}} = 1.4105 \times 10^{-4} \text{ m}^2$$

$$D := \sqrt[\pi]{\frac{4 \cdot A}{\pi}} = 0.0134 \text{ m}$$

```
\[
\sigma = \rho \cdot \frac{4\pi \cdot A^2}{\lambda^2} = 1 \times 10^6 \text{ m}^2 \times \frac{(0.5 \times 10^{-6} \text{ m})^2}{(4 \times \pi \times 1)} = 1.4105 \times 10^{-4} \text{ m}^2
\]

\[
A := \sqrt[\lambda^2]{\frac{\sigma \cdot \lambda^2}{4\pi \cdot \rho}} = \sqrt[\lambda^2]{\frac{1 \times 10^6 \text{ m}^2 \times (0.5 \times 10^{-6} \text{ m})^2}{(4 \times \pi \times 1)}} = 1.4105 \times 10^{-4} \text{ m}^2
\]

\[
D := \sqrt[\pi]{\frac{4 \cdot A}{\pi}} = \sqrt[\pi]{\frac{4 \cdot (1.4105 \times 10^{-4} \text{ m}^2)}{\pi}} = 0.0134 \text{ m}
\]
```
Exercise 9-1  A radar warning receiver (RWR) has the following characteristics: antenna gain in the direction of the radar $G_{\text{RWR}} = 3$ dBi and losses $L_{\text{RWR}} = 9$ dB. A radar system has the following characteristics: peak transmit power $P_R = 500$ kW, antenna gain in the direction of the target/RWR $G_{\text{RT}} = 35$ dBi, carrier frequency $f_c = 5$ GHz, and radar transmit loss $L_{\text{Rt}} = 2$ dB. The target/RWR is at a radar-to-target/RWR range $R_{\text{RT}} = 150$ km. Determine the following: a) the transmitted radar effective radiated power, $E_{\text{PR}}$ (Watts and dBW), b) the received radar power density at the RWR antenna, and c) the received single pulse radar signal power, $S_{\text{RWR}}$ (Watts and dBW). Compare the received single pulse radar signal power to the radar's transmitted effective radiated power.

We start by defining the values for all the terms and necessary constants.

$$
G_{\text{RWR}} \text{ dBi} = 3 \quad L_{\text{RWR}} \text{ dB} = 9 \quad P_R = 500 \times 10^3 \text{ W} \quad G_{\text{RT}} \text{ dBi} = 35 \quad f_c = 5 \times 10^9 \text{ Hz} \quad L_{\text{Rt}} \text{ dB} = 2 \quad R_{\text{RT}} = 150 \times 10^3 \text{ m} \quad c = 3 \times 10^8 \frac{\text{m}}{\text{sec}}
$$

Convert values from dB to absolute as we will be using algebraic equations.

$$
G_{\text{RWR}} := 10 \left( \frac{G_{\text{RWR}} \text{ dBi}}{10} \right) = 1.9953 \\
L_{\text{RWR}} := 10 \left( \frac{L_{\text{RWR}} \text{ dB}}{10} \right) = 7.9433 \\
G_{\text{RT}} := 10 \left( \frac{G_{\text{RT}} \text{ dBi}}{10} \right) = 3.1623 \times 10^3
$$

$$
L_{\text{Rt}} := 10 \left( \frac{L_{\text{Rt}} \text{ dB}}{10} \right) = 1.5849
$$

a) Compute the transmitted effective radiated power (ERP) of the radar, $E_{\text{PR}}$ (Watts) and $E_{\text{PR, dBW}}$ (dBW), using Equation 9-1.
\[ \text{ERPR} := \frac{P_R \cdot G_{RT}}{L_{Rt}} = 9.9763 \times 10^8 \text{W} \]

\[ = \frac{500 \cdot 10^3 \cdot W \cdot 10^\left(\frac{35}{10}\right)}{10^\left(\frac{2}{10}\right)} = 9.9763 \times 10^8 \text{W} \]

\[ \text{ERPR\_dBW} := 10 \cdot \log \left( \frac{\text{ERPR}}{1\text{W}} \right) = 89.9897 \]

\[ = 10 \cdot \log \left( \frac{9.9763 \cdot 10^8 \cdot W}{1\cdot W} \right) = 89.9897 \]

b) Compute the received radar power density at the RWR antenna using Equation 9-26.

\[ \frac{P_R \cdot G_{RT}}{(4\pi) \cdot R_{RT}^2 \cdot L_{Rt}} = 3.5284 \times 10^{-3} \frac{\text{W}}{\text{m}^2} \]

\[ = \frac{500 \cdot 10^3 \cdot W \cdot 10^\left(\frac{35}{10}\right)}{4 \cdot \pi \cdot (150 \cdot 10^3 \cdot \text{m})^2 \cdot 10^\left(\frac{2}{10}\right)} = 3.5284 \times 10^{-3} \frac{\text{W}}{\text{m}^2} \]

c) Compute the received single pulse radar signal power, \( S_{\text{RWR}} \) (Watts) and \( S_{\text{RWR\_dBW}} \) (dBW), using Equation 9-5. Since we will need the wavelength, \( \lambda \) (meters), we will compute it first using Equation 2-2.

\[ \lambda := \frac{c}{f_c} = 0.06 \text{m} \]

\[ = \frac{3 \cdot 10^8 \cdot \left( \frac{\text{m}}{\text{sec}} \right)}{(5 \cdot 10^9 \cdot \text{Hz})} = 0.06 \text{m} \]
To compare the received single pulse radar signal power to the radar ERP we compute the ratio of the two. The RWR received approximately 15 orders of magnitude (power of 10) less power than the radar it transmitted. The vast majority of the power is lost by the \( \frac{1}{R^2} \) propagation from the radar system to the RWR, here a 150 km propagation path length.

\[
\frac{S_{RWR}}{\text{ERP}_R} = 4.0337 \times 10^{-16}
\]

\[
10 \log \left( \frac{S_{RWR}}{\text{ERP}_R} \right) = -153.943
\]
Exercise 9-2  The radar warning receiver (RWR) from Exercise 9-1 has the following additional characteristics: minimum discernible signal sensitivity \( S_{\text{min\_dis}} = -80 \text{ dBW} \) and receiver bandwidth \( B_{\text{RWR}} = 6 \text{ GHz} \). What is the receiver noise figure \( F_{\text{RWR}} \) (no units and dB)? What is the single pulse radar signal-to-noise ratio of the RWR, \( (S/N)_{\text{RWR}} \) (no units and dB)? [Hint: use the results from Exercise 9-1]

We start by defining the values for all the terms and necessary constants.

\[
S_{\text{min\_dis\_dBW}} = -80 \quad B_{\text{RWR}} = 6 \times 10^9 \text{ Hz} \quad S_{\text{RWR}} = 4.0241 \times 10^{-7} \text{ W} \quad T_0 = 290 \text{ K} \quad k = 1.38 \times 10^{-23} \text{ W} \cdot \text{sec} \cdot \text{K}^{-1}
\]

Convert values from dB to absolute as we will be using algebraic equations.

\[
S_{\text{min\_dis}} := 10^{(-80/10)} \cdot 1\text{W} = 1 \times 10^{-8} \text{W}
\]

We will compute the RWR noise figure, \( F_{\text{RWR}} \) (no units) and \( F_{\text{RWR\_dB}} \) (dB), in multiple steps. First, we know the RWR thermal noise power, \( N_{\text{RWR}} \) (Watts), is a function of the RWR noise figure, as given in Equation 9-8. We can solve this equation for the RWR noise figure. Second, we know the minimum discernible signal sensitivity, \( S_{\text{min\_dis}} \) (Watts), equals the RWR thermal noise power, as given in Equation 9-15. With these two relationships we can compute the RWR noise figure. All these equations and calculations are shown below.

\[
N_{\text{RWR}} = F_{\text{RWR}} \cdot k \cdot T_0 \cdot B_{\text{RWR}}
\]

\[
F_{\text{RWR}} = \frac{N_{\text{RWR}}}{k \cdot T_0 \cdot B_{\text{RWR}}}
\]

\[
S_{\text{min\_dis}} = N_{\text{RWR}}
\]

\[
N_{\text{RWR}} := S_{\text{min\_dis}} = 1 \times 10^{-8} \text{W}
\]

\[
F_{\text{RWR}} := \frac{N_{\text{RWR}}}{k \cdot T_0 \cdot B_{\text{RWR}}} = 416.4584
\]

\[
F_{\text{RWR\_dB}} := 10 \log(F_{\text{RWR}}) = 26.1957
\]

\[
10 \log \left( \frac{1 \times 10^{-8} \text{W}}{1.38 \times 10^{-23} \text{ W} \cdot \text{sec} \cdot \text{K}^{-1} \cdot 290 \text{ K} \cdot 6 \times 10^9 \text{ Hz}} \right) = 416.4584
\]

\[
= 10 \log (416.45843744794263) = 26.1957
\]
The single-pulse radar signal-to-noise ratio of the RWR, $\text{SNR}_{\text{RWR}}$ (no units) and $\text{SNR}_{\text{RWR}}_{\text{dB}}$ (dB), is given in Equation 9-11. We computed the single-pulse radar signal power, $S_{\text{RWR}}$ (Watts), in Exercise 9-1, and will use it here. The calculations are shown below.

\[
\text{SNR}_{\text{RWR}} := \frac{S_{\text{RWR}}}{N_{\text{RWR}}} = 40.241
\]

\[
= \frac{4.0241 \times 10^{-7} \text{W}}{1 \times 10^{-8} \text{W}} = 40.241
\]

\[
\text{SNR}_{\text{RWR}}_{\text{dB}} := 10 \log (\text{SNR}_{\text{RWR}}) = 16.0467
\]

\[
= 10 \log (40.240999999999993) = 16.0467
\]
**Exercise 9-3** The radar warning receiver (RWR) from Exercises 9-1 and 9-2 has a signal-to-noise ratio required for detection (detection threshold) \( SNR_{dtRWR} = 15 \, \text{dB} \). What is the RWR detection range, \( R_{dtRWR} \) (meters)? [Hint: use the results from Exercises 9-1 and 9-2]

We start by defining the values for all the terms and necessary constants.

\[
\begin{align*}
G_{RWR\_dBi} & \equiv 3 & L_{RWR\_dB} & \equiv 9 & S_{\text{min\_dis\_dBW}} & \equiv -80 & F_{RWR} & \equiv 416.4584 \\
SNR_{dtRWR\_dB} & \equiv 15 & P_R & \equiv 500 \times 10^3 \cdot \text{W} & G_{RT\_dBi} & \equiv 35 & f_c & \equiv 5 \times 10^9 \cdot \text{Hz} & B_{RWR} & \equiv 6 \times 10^9 \cdot \text{Hz} \\
c & \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}} & T_0 & \equiv 290 \cdot \text{K} & k & \equiv 1.38 \times 10^{-23} \cdot \frac{\text{W} \cdot \text{sec}}{\text{K}}
\end{align*}
\]

Convert values from dB to absolute as we will first be using algebraic equations.

\[
\begin{align*}
G_{RWR} & := 10^{\left(\frac{G_{RWR\_dBi}}{10}\right)} = 1.9953 \\
L_{RWR} & := 10^{\left(\frac{L_{RWR\_dB}}{10}\right)} = 7.9433 \\
G_{RT} & := 10^{\left(\frac{G_{RT\_dBi}}{10}\right)} = 3.1623 \times 10^3 \\
SNR_{dtRWR} & := 10^{\left(\frac{SNR_{dtRWR\_dB}}{10}\right)} = 31.6228 \\
S_{\text{min\_dis\_dBW}} & := 10^{\left(\frac{S_{\text{min\_dis\_dBW}}}{10}\right)} \cdot 1 \cdot \text{W} = 1 \times 10^{-8} \cdot \text{W}
\end{align*}
\]

Since we were given a wide range of radar and RWR characteristics for this exercise, and Exercises 9-1 and 9-2, we can compute the RWR detection range, \( R_{dtRWR} \) (meters), using multiple equations. First, we will use Equation 9-16. Since we need the wavelength, \( \lambda \) (meters), we will compute it using Equation 2-2.
\[ \lambda := \frac{c}{f_c} = 0.06 \text{m} \]

\[ \frac{3 \times 10^8 \text{ m}}{5 \times 10^9 \text{ Hz}} = 0.06 \text{m} \]

\[
\sqrt{\frac{P_R \cdot G_{RT} \cdot G_{RWR} \cdot \lambda^2}{(4\pi)^2 \cdot \text{SNR}_{dtRWR} \cdot F_{RWR} \cdot k \cdot T_0 \cdot B_{RWR} \cdot L_{RWR}}} = 1.6921 \times 10^5 \text{ m}
\]

\[
\left( \frac{500 \times 10^3 \text{ W}}{10^{10}} \right) \left[ 10 \left( \frac{35}{10} \right) \right] \left[ 10 \left( \frac{3}{10} \right) \right] \left( \frac{3 \times 10^8 \text{ m}}{5 \times 10^9 \text{ Hz}} \right)^2 \left( \frac{1.38 \times 10^{-23} \text{ W s K}^{-1}}{416.4584} \right) \left( 290 \text{ K} \right) \left( 6 \times 10^9 \text{ Hz} \right) \left[ 10 \left( \frac{9}{10} \right) \right] = 1.6921 \times 10^5 \text{ m}
\]

We can use the logarithmic form of this equation as well, Equation 9-18. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the RWR detection range.

\[
\text{term}_1 := 10 \log \left( \frac{P_R}{1 \text{ W}} \right) + G_{RT \_dB} + G_{RWR \_dB} - 20 \log \left( \frac{f_c}{1 \text{ Hz}} \right) = -98.9897
\]

\[
= 10 \log \left( \frac{500 \times 10^3 \text{ W}}{1 \cdot \text{W}} \right) + 35 + 3 - 20 \log \left( \frac{5 \times 10^9 \text{ Hz}}{1 \cdot \text{Hz}} \right) = -98.9897
\]

\[
\text{term}_2 := -\text{SNR}_{dtRWR \_dB} - 10 \log \left( F_{RWR} \right) - 10 \log \left( \frac{B_{RWR}}{1 \text{Hz}} \right) - L_{RWR \_dB} = -147.9772
\]

\[
= -15 - 10 \log (416.4584) - 10 \log \left( \frac{6 \times 10^9 \text{ Hz}}{1 \cdot \text{Hz}} \right) - 9 = -147.9772
\]
\[
\frac{\left( \text{term}_1 + \text{term}_2 + 351.5355 \right)}{20} \cdot m = 1.6921 \times 10^5 \text{m}
\]

\[
= 10^{-20} \left( -98.989700043360187 + 147.97722876648047 + 351.5355 \right) \cdot m = 1.6921 \times 10^5 \text{m}
\]

Next we use Equation 9-19, based on the minimum detectable signal sensitivity, \( S_{\text{min,dt}} \) (Watts). We will compute the minimum detectable signal sensitivity using Equation 9-14.

\[
N_{\text{RWR}} := F_{\text{RWR}} \cdot k \cdot T_0 \cdot B_{\text{RWR}} = 10 \times 10^{-9} \text{W}
\]

\[
= 416.4584 \cdot 1.38 \cdot 10^{-23} \cdot \left( \frac{\text{W} \cdot \text{sec}}{\text{K}} \right) \cdot (290 \cdot \text{K}) \cdot (6 \cdot 10^9 \cdot \text{Hz}) = 10 \times 10^{-9} \text{W}
\]

\[
S_{\text{min,dt}} := N_{\text{RWR}} \cdot \text{SNR}_{\text{dt, RWR}} = 3.1623 \times 10^{-7} \text{W}
\]

\[
= 10 \cdot 10^{-9} \cdot \text{W} \cdot 31.622776601683793 = 3.1623 \times 10^{-7} \text{W}
\]

\[
10 \log \left( \frac{S_{\text{min,dt}}}{1 \text{W}} \right) = -65
\]

\[
= 10 \cdot \log \frac{3.1623 \cdot 10^{-7} \text{W}}{(1 \cdot \text{W})} = -65
\]

\[
\sqrt{\frac{P_R \cdot G_{\text{RT}} \cdot G_{\text{RWR}} \cdot \lambda^2}{(4 \pi)^2 \cdot S_{\text{min,dt}} \cdot L_{\text{RWR}}}} = 1.6921 \times 10^5 \text{m}
\]

\[
= \sqrt{\frac{500 \cdot 10^3 \cdot \text{W} \cdot 10 \cdot \left( \frac{35}{10} \right) \cdot 10 \cdot \left( \frac{3}{10} \right) \cdot \left[ 3.108 \cdot \left( \frac{\text{m}}{\text{sec}} \right) \right]^2}{(4 \pi)^2 \cdot (3.1623 \cdot 10^{-7} \text{W}) \cdot 10 \cdot \left( \frac{9}{10} \right)}} = 1.6921 \times 10^5 \text{m}
\]
We can use the logarithmic form of this equation as well, Equation 9-21. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the RWR detection range.

\[
\text{term}_3 := 10 \log \left( \frac{P_R}{1W} \right) + G_{RT\_dBi} + G_{RWR\_dBi} - 20 \log \left( \frac{f_c}{\text{1Hz}} \right) = -98.9897
\]

\[
= 10 \cdot \log \left( \frac{500 \cdot 10^3 \cdot W}{(1 \cdot \text{W})} \right) - 20 \cdot \log \left( \frac{5 \cdot 10^9 \cdot \text{Hz}}{(1 \cdot \text{Hz})} \right) = -98.9897
\]

\[
\text{term}_4 := -10 \log \left( \frac{S_{\text{min\_dt}}}{1W} \right) - L_{RWR\_dB} + 147.5582 = 203.5582
\]

\[
= (-10) \cdot \log \left( \frac{3.1623 \cdot 10^{-7} \cdot W}{(1 \cdot \text{W})} \right) - 9 + 147.5582 = 203.5582
\]

\[
\left( \text{term}_3 + \text{term}_4 \right) \cdot m = 1.6921 \times 10^5 \text{m}
\]

\[
= 10^{\frac{(-98.989700043360187+203.55820039051761)}{20}} \cdot m = 1.6921 \times 10^5 \text{m}
\]

And finally, we use Equation 9-22, based on the minimum discernable signal sensitivity, \( S_{\text{min\_dis}} \) (Watts).

\[
\sqrt{\frac{P_R \cdot G_{RT} \cdot G_{RWR} \cdot \lambda^2}{(4 \pi)^2 \cdot \text{SNR}_{dt\_RWR} \cdot S_{\text{min\_dis}} \cdot L_{RWR}}} = 1.6921 \times 10^5 \text{m}
\]
We can use the logarithmic form of this equation as well, Equation 9-24. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the RWR detection range.

\[
\text{term}_5 := 10 \log \left( \frac{P_R}{1\text{W}} \right) + G_{RT}_{\text{dBi}} + G_{RWR}_{\text{dBi}} - 20 \log \left( \frac{f_c}{1\text{Hz}} \right) = -98.9897
\]

\[
= 10 \cdot \log \left( \frac{500 \cdot 10^3 \text{W}}{1 \text{W}} \right) + 35 + 3 - 20 \cdot \log \left( \frac{5 \cdot 10^9 \text{Hz}}{1 \text{Hz}} \right) = -98.9897
\]

\[
\text{term}_6 := -\text{SNR}_{\text{dtRWR}} - S_{\text{min} \text{ dis} \text{ dBW}} - L_{\text{RWR}} + 147.5582 = 203.5582
\]

\[
= -15 - 80 - 9 + 147.5582 = 203.5582
\]

\[
\frac{\left( \text{term}_5 + \text{term}_6 \right)}{10^{20}} \cdot \text{m} = 1.6921 \times 10^5 \text{m}
\]

\[
= 10^{\left( \frac{-98.98970043360187 + 203.5582}{20} \right)} \cdot \text{m} = 1.6921 \times 10^5 \text{m}
\]
**Exercise 9-4** The first sidelobe of the radar antenna is pointing at the RWR from Exercises 9-1, 9-2, and 9-3. The radar antenna has a uniform illumination function (see Section 4.1 and Table 4-1). What is the RWR detection range, $R_{dRWR}$ (meters)? [Hint: use the results from Exercises 9-1, 9-2, and 9-3]

We start by defining the values for all the terms and necessary constants.

$G_{RWR\_dBi} = 3$  
$L_{RWR\_dB} = 9$  
$S_{min\_dis\_dBW} = -80$  
$F_{RWR} = 416.4584$  
$B_{RWR} = 6 \times 10^9 \cdot Hz$

$SNR_{dtRWR\_dB} = 15$  
$P_R = 500 \times 10^3 \cdot W$  
$f_C = 5 \times 10^9 \cdot Hz$

$c = 3 \times 10^8 \cdot \frac{m}{sec}$  
$T_0 = 290 \cdot K$  
$k = 1.38 \times 10^{-23} \cdot W \cdot sec \cdot K$

We first need to determine the radar antenna gain in the direction of the RWR, $G_{RT\_dBi}$ (dBi). Using the discussion of antenna sidelobes in Section 4.1 and the values in Table 4-1, we find for a uniform illumination function the first sidelobe is 13.2 dB down from the mainbeam gain. Thus, we compute the antenna gain of first sidelobe of the radar antenna as shown below.

$G_{RT\_dBi} = 35 - 13.2 = 21.8$

Convert values from dB to absolute as we will first be using algebraic equations.

$G_{RWR} := 10^{\left( \frac{G_{RWR\_dBi}}{10} \right)} = 1.9953$  
$L_{RWR} := 10^{\left( \frac{L_{RWR\_dB}}{10} \right)} = 7.9433$  
$G_{RT} := 10^{\left( \frac{G_{RT\_dBi}}{10} \right)} = 151.3561$

$SNR_{dtRWR} := 10^{\left( \frac{SNR_{dtRWR\_dB}}{10} \right)} = 31.6228$  
$S_{min\_dis} := 10^{\left( \frac{S_{min\_dis\_dBW}}{10} \right)} \cdot 1W = 1 \times 10^{-8} W$
Since we were given a wide range of radar and RWR characteristics for this exercise, and Exercises 9-1 thru 9-3, we can compute the RWR detection range, \( R_{dRWR} \) (meters), using multiple equations. First, we will use Equation 9-16. Since we need the wavelength, \( \lambda \) (meters), we will compute it using Equation 2-2.

\[
\lambda := \frac{c}{f_c} = 0.06 \text{m}
\]

\[
\sqrt{\frac{P_R \cdot G_{RT} \cdot G_{RWR} \cdot \lambda^2}{(4\pi)^2 \cdot \text{SNR}_{dtRWR} \cdot F_{RWR} \cdot k \cdot T_0 \cdot B_{RWR} \cdot L_{RWR}}} = 3.7019 \times 10^4 \text{m}
\]

\[
\left(\frac{500 \times 10^3 \cdot W}{10}\right) \left[10^3 \cdot 10\right] \left[10^2 \cdot \frac{3 \times 10^8 \cdot \text{m}}{5 \times 10^9 \cdot \text{Hz}}\right]^2 \left[\left(\frac{15}{10}\right)\right] \left(\frac{1.38 \times 10^{-23} \cdot \text{W} \cdot \text{sec}}{K}\right) \left(290 \cdot K\right) \left(6 \times 10^9 \cdot \text{Hz}\right) \left[\left(\frac{9}{10}\right)\right] = 3.7019 \times 10^4 \text{m}
\]

We can use the logarithmic form of this equation as well, Equation 9-18. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the RWR detection range.

\[
\text{term}_1 := 10 \log \left(\frac{P_R}{1 \cdot W}\right) + G_{RT\_dbi} + G_{RWR\_dbi} - 20 \log \left(\frac{f_c}{1 \cdot \text{Hz}}\right) = -112.1897
\]

\[
= 10 \cdot \log \left[\frac{500 \times 10^3 \cdot W}{1 \cdot W}\right] + 21.8 + 3 - 20 \cdot \log \left[\frac{5 \times 10^9 \cdot \text{Hz}}{1 \cdot \text{Hz}}\right] = -112.1897
\]
Next we use Equation 9-19, based on the minimum detectable signal sensitivity, $S_{\text{min}_d}$ (Watts). We will compute the minimum detectable signal sensitivity using Equation 9-14.

$$N_{\text{RWR}} := F_{\text{RWR}} \cdot k \cdot T_0 \cdot B_{\text{RWR}} = 10 \times 10^{-9} \text{W}$$

$$= 416.4584 \cdot \frac{1.38 \cdot 10^{-23} \cdot (\text{W} \cdot \text{sec})}{\text{K}} \cdot (290 \cdot \text{K}) \cdot (6 \cdot 10^9 \cdot \text{Hz}) = 10 \times 10^{-9} \text{W}$$

$$S_{\text{min}_d} := N_{\text{RWR}} \cdot \text{SNR}_{dt,RWR} = 3.1623 \times 10^{-7} \text{W}$$

$$= 10 \cdot 10^{-9} \cdot \text{W} \cdot 31.622776601683793 = 3.1623 \times 10^{-7} \text{W}$$

$$10 \log \left( \frac{\text{S}_{\text{min}_d}}{1 \text{W}} \right) = -65$$

$$= 10 \cdot \log \left[ \frac{3.1623 \cdot 10^{-7} \text{W}}{1 \text{W}} \right] = -65$$

$$\sqrt{\frac{P_{\text{R}} \cdot G_{\text{RT}} \cdot G_{\text{RWR}} \cdot \chi^2}{(4\pi)^2 \cdot S_{\text{min}_d} \cdot L_{\text{RWR}}}} = 3.7019 \times 10^4 \text{m}$$
$$= \sqrt{\frac{500 \cdot 10^3 \cdot W \cdot 10 \left( \frac{21.8}{10} \right) \cdot 10 \left( \frac{3}{10} \right) \cdot (0.06 \cdot m)^2}{(4 \cdot \pi)^2 \cdot \left( 3.1623 \cdot 10^{-7} \cdot W \right) \cdot 10 \left( \frac{9}{10} \right)}} = 3.7019 \times 10^4 \text{ m}$$

We can use the logarithmic form of this equation as well, Equation 9-21. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the RWR detection range.

$$\text{term}_3 := 10 \log \left( \frac{P_R}{1 \text{ W}} \right) + G_{\text{RT}_\text{dBi}} + G_{\text{RWR}_\text{dBi}} - 20 \log \left( \frac{f_c}{1 \text{ Hz}} \right) = -112.1897$$

$$= 10 \cdot \log \left( \frac{500 \cdot 10^3 \cdot W}{(1 \cdot \text{ W})} \right) + 21.8 + 3 - 20 \cdot \log \left( \frac{5 \cdot 10^9 \cdot \text{ Hz}}{(1 \cdot \text{ Hz})} \right) = -112.1897$$

$$\text{term}_4 := -10 \log \left( \frac{S_{\text{min}_\text{dis}}}{1 \text{ W}} \right) - L_{\text{RWR}_\text{dB}} + 147.5582 = 203.5582$$

$$= (-10) \cdot \log \left( \frac{3.1623 \cdot 10^{-7} \cdot W}{(1 \cdot \text{ W})} \right) - 9 + 147.5582 = 203.5582$$

$$\frac{\left( \text{term}_3 + \text{term}_4 \right)}{10} \cdot m = 3.7019 \times 10^4 \text{ m}$$

$$= 10 \cdot \log \left( \frac{(-112.18970004336019 + 203.55820039051761)}{20} \right) \cdot m = 3.7019 \times 10^4 \text{ m}$$

And finally, we use Equation 9-22, based on the minimum discernable signal sensitivity, $S_{\text{min}_\text{dis}}$ (Watts).
\[
\frac{\sqrt{P_R \cdot G_{RT} \cdot G_{RWR} \cdot \lambda^2}}{(4\pi)^2 \cdot SNR_{dRWR} \cdot S_{min\_dis} \cdot LRWR} = 3.7019 \times 10^4 \text{m}
\]

We can use the logarithmic form of this equation as well, Equation 9-24. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the RWR detection range.

\[
\text{term}_5 := 10\log\left(\frac{P_R}{1\text{W}}\right) + G_{RT\_dBi} + G_{RWR\_dBi} - 20\log\left(\frac{f_c}{1\text{Hz}}\right) = -112.1897
\]

\[
= 10 \cdot \log\left[\frac{500 \cdot 10^3 \cdot \text{W}}{(1\cdot\text{W})}\right] + 21.8 + 3 - 20 \cdot \log\left[\frac{5 \cdot 10^9 \cdot \text{Hz}}{(1\cdot\text{Hz})}\right] = -112.1897
\]

\[
\text{term}_6 := -\text{SNR}_{dRWR\_dB} - S_{min\_dis\_dB} - LRWR\_dB + 147.5582 = 203.5582
\]

\[
= -15 - 80 - 9 + 147.5582 = 203.5582
\]

\[
\frac{\text{term}_5 + \text{term}_6}{20} \cdot \text{m} = 3.7019 \times 10^4 \text{m}
\]

Notice how much small the RWR detection range is when the first sidelobe of the antenna is pointing at the RWR; compare the results of this exercise with those in Exercise 9-3.
**Exercise 9-5** A radar warning receiver (RWR) has the following characteristics: antenna gain in the direction of the radar \( G_{RWR} = 4 \) dBi and losses \( L_{RWR} = 8 \) dB. The RWR receives a single pulse radar signal peak power \( S_{RWR} = -60 \) dBW. The RWR detects the radar pulse, extracts the radar carrier frequency \( f_c = 10 \) GHz, identifies the radar system, and looks up the following radar characteristics from the Emitter Identification table: peak transmit power \( P_R = 100 \) kW and mainbeam antenna gain \( G_{RT} = 38 \) dBi. What is the estimated radar-to-target/RWR range, \( R_{RT} \) (meters)?

We start by defining the values for all the terms and necessary constants.

\[
\begin{align*}
G_{RWR\_dBi} &= 4 & L_{RWR\_dB} &= 8 & S_{RWR\_dBW} &= -60 & f_c &= 10 \times 10^9 \text{ Hz} & P_R &= 100 \times 10^3 \text{ W} \\
G_{RT\_dBi} &= 38 & c &= 3 \times 10^8 \frac{\text{m}}{\text{sec}}
\end{align*}
\]

Convert values from dB to absolute as we will be using algebraic equations.

\[
\begin{align*}
S_{RWR} &= 10 \left( \frac{S_{RWR\_dBW}}{10} \right) \cdot 1\text{W} = 1 \times 10^{-6} \text{W} \\
L_{RWR} &= 10 \left( \frac{L_{RWR\_dB}}{10} \right) = 6.3096 \\
G_{RWR} &= 10 \left( \frac{G_{RWR\_dBi}}{10} \right) = 2.5119 \\
G_{RT} &= 10 \left( \frac{G_{RT\_dBi}}{10} \right) = 6.3096 \times 10^3
\end{align*}
\]

We can compute the estimated radar-to-target/RWR range, \( R_{RT} \) (meters), using Equation 9-26. Since we need the wavelength, \( \lambda \) (meters), we will compute it using Equation 2-2.
\[ \lambda := \frac{c}{f_c} = 0.03 \text{m} \]

\[
= \frac{3 \times 10^8 \left( \frac{\text{m}}{\text{sec}} \right)}{(10 \times 10^9 \cdot \text{Hz})} = 0.03 \text{m}
\]

\[ R_{RT} := \sqrt{\frac{P_{RT} \cdot G_{RT} \cdot G_{RWR} \cdot \lambda^2}{(4\pi)^2 \cdot S_{RWR} \cdot L_{RWR}}} = 3.7837 \times 10^4 \text{m} \]

\[ R_{RT} = 37.8365 \cdot \text{km} \]

\[
= \sqrt{\frac{100 \cdot 10^3 \cdot \text{W} \cdot \frac{38}{10} \cdot \frac{4}{10} \cdot \left[ \frac{3 \times 10^8 \left( \frac{\text{m}}{\text{sec}} \right)}{(10 \times 10^9 \cdot \text{Hz})} \right]^2}{(4\pi)^2 \cdot \frac{(-60)}{10} \cdot \text{W} \cdot \frac{8}{10}}} = 3.7837 \times 10^4 \text{m}
\]
Exercise 10-1  A self-protection jammer has the following characteristics: peak transmit power \( P_J = 25 \) Watts, transmit antenna gain in the direction of the radar \( G_{JR} = 3 \) dBi, total jammer-related losses \( L_J = 15 \) dB, and jammer transmit loss \( L_{Jt} = 2 \) dB. The jammer is on the target and jamming the radar system from Exercises 2-5 and 3-15. Compute the following: (a) the transmitted jammer effective radiated power, ERP\(_J\) (Watts and dBW), (b) the received jammer power density at the radar receive antenna, and (c) the received jammer power, \( J \) (Watts and dBW). Compare the received jammer power to the jammer transmitted effective radiated power. [Hint: use the results from Exercises 2-5 and 3-15]

We start by defining the values for all the terms and necessary constants. We also define the values for all the necessary terms from Exercises 2-5 and 3-15. Convert from dB to absolute as needed.

\[
P_J = 25 \, \text{W} \quad G_{JR_{\text{dBi}}} = 3 \quad L_{J_{\text{dB}}} = 15 \quad L_{Jt_{\text{dB}}} = 2 \quad G_{RT_{\text{dBi}}} = 38 \quad f_c = 3 \times 10^9 \, \text{Hz}
\]

\[
R_{RT} = 150 \times 10^3 \, \text{m}
\]

\[
G_{JR} := 10^\left(\frac{G_{JR_{\text{dBi}}}}{10}\right) = 1.9953
\]

\[
L_J := 10^\left(\frac{L_{J_{\text{dB}}}}{10}\right) = 31.6228
\]

\[
L_{Jt} := 10^\left(\frac{L_{Jt_{\text{dB}}}}{10}\right) = 1.5849
\]

\[
G_{RT} := 10^\left(\frac{G_{RT_{\text{dBi}}}}{10}\right) = 6.3096 \times 10^3
\]

(a) Compute the transmitted effective radiated power (ERP) of the self-protection jammer, ERP\(_J\) (Watts) and ERP\(_J_{\text{dBW}}\) (dBW), as given in Equation 10-1.
ERP_J := \frac{P_J \cdot G_{JR}}{L_{Jt}} = 31.4731 W

\[\begin{align*}
\text{ERP}_J_{\text{dBW}} := 10 \log \left( \frac{\text{ERP}_J}{1 W} \right) &= 14.9794
\end{align*}\]

\[\begin{align*}
\text{ERP}_{J_{\text{dBW}}} &= 10 \log \left( \frac{31.4731 \cdot W}{1 W} \right) = 14.9794
\end{align*}\]

(b) Compute the self-protection jammer power density at the radar receive antenna, as given in Equation 10-2.

\[\frac{P_J \cdot G_{JR}}{4 \pi \cdot R_{RT}^2 \cdot L_{Jt}} = 1.1131 \times 10^{-10} \frac{W}{m^2}\]

\[\begin{align*}
\frac{25 \cdot W \cdot 10^\left(\frac{3}{10}\right)}{4 \cdot \pi \cdot \left(150 \cdot 10^3 \cdot m \right)^2 \cdot 10^\left(\frac{2}{10}\right)} &= 1.1131 \times 10^{-10} \frac{W}{m^2}
\end{align*}\]

(c) Compute the received self-protection jammer power, J (Watts) and J_{dBW} (dBW), in the radar receiver, as given in Equation 10-5. We will need the wavelength, \(\lambda\) (meters), and will compute it using Equation 2-2.

\[\lambda := \frac{c}{f_c} = 0.1 m\]

\[\begin{align*}
\lambda &= \frac{3 \cdot 10^8 \cdot \frac{m}{sec}}{3 \cdot 10^9 \cdot Hz} = 0.1 m
\end{align*}\]

\[\begin{align*}
J := \frac{P_J \cdot G_{JR} \cdot G_{RT} \cdot \lambda^2}{(4 \pi)^2 \cdot R_{RT}^2 \cdot L_J} = 2.8012 \times 10^{-11} W
\end{align*}\]
\[
J_{\text{dBW}} := 10 \cdot \log \left( \frac{J}{1\text{W}} \right) = -105.5266
\]

To compare the received self-protection jammer power to the self-protection jammer's transmitted ERP we compute the ratio of the two. The radar received approximately 12 orders of magnitude (power of 10) less power than the jammer transmitted. The vast majority of the power is lost by the \(1/R^2\) propagation from the jammer to the radar, here a 150 km propagation path length.

\[
\frac{J}{\text{ERP}_J} = 8.9002 \times 10^{-13}
\]

\[
10 \log \left( \frac{J}{\text{ERP}_J} \right) = -120.506
\]
Exercise 10-2: The self-protection jammer from Exercise 10-1 uses a noise waveform with a bandwidth $B_J = 200$ MHz centered about the radar carrier frequency. Compute the following: (a) the jammer noise power in the radar receiver, $J_N$ (Watts and dBW), (b) the interference signal power, $I$ (Watts and dBW), and (c) the target signal-to-interference ratio after integration of multiple pulses, $(S/I)_n$ (no units and dB). Compare the target signal-to-interference ratio with the target signal-to-noise ratio after integration of multiple pulses from Exercise 3-15. [Hint: use the results of Exercises 2-5 and 3-15]

We start by defining the values for all the terms and necessary constants.

$$
B_J = 200 \times 10^6 \cdot \text{Hz} \quad B_R = 750 \times 10^3 \cdot \text{Hz} \quad G_{sp} = 1 \quad G_I = 9.745
$$

$$
N = 1.1949 \times 10^{-14} \cdot \text{W} \quad S = 7.9445 \times 10^{-14} \cdot \text{W} \quad J = 2.8012 \times 10^{-11} \cdot \text{W}
$$

(a) Compute the self-protection jammer noise power in the radar receiver, $J_N$ (Watts) and $J_{N\_dBW}$ (dBW), as given in Equation 10-13.

$$
J_N := J \cdot \left( \frac{B_R}{B_J} \right) = 1.0504 \times 10^{-13} \cdot \text{W} \quad \Rightarrow \quad = 2.8012 \times 10^{-11} \cdot \text{W} \cdot \frac{750 \times 10^3 \cdot \text{Hz}}{200 \times 10^6 \cdot \text{Hz}} = 1.0504 \times 10^{-13} \cdot \text{W}
$$

$$
J_{N\_dBW} := 10 \cdot \log \left( \frac{J_N}{1 \cdot \text{W}} \right) = -129.7862 \quad \Rightarrow \quad = 10 \cdot \log \left( \frac{1.0504 \times 10^{-13} \cdot \text{W}}{1 \cdot \text{W}} \right) = -129.7865
$$

(b) Compute the interference signal power, $I$ (Watts) and $I\_dBW$ (dBW), as given in Equation 10-14.

$$
I := N + J_N = 1.1699 \times 10^{-13} \cdot \text{W} \quad \Rightarrow \quad = 1.1949 \times 10^{-14} \cdot \text{W} + 1.0504 \times 10^{-13} \cdot \text{W} = 1.1699 \times 10^{-13} \cdot \text{W}
$$

$$
I\_dBW := 10 \cdot \log \left( \frac{I}{1 \cdot \text{W}} \right) = -129.3184 \quad \Rightarrow \quad = 10 \cdot \log \left( \frac{1.1699 \times 10^{-13} \cdot \text{W}}{1 \cdot \text{W}} \right) = -129.3185
$$

(c) Compute the multiple pulse signal-to-interference ratio, $(S/I)_n$ (no units) and $(S/I)_{n\_dB}$ (dB), after non-coherent integration, as given in Equation
10-17.

\[
SIR_n := \frac{S \cdot G_{sp} \cdot G_I}{N + J_N} = \frac{7.9445 \cdot 10^{-14} \cdot W \cdot 1 \cdot 9.75}{1.1949 \cdot 10^{-14} \cdot W + 1.0504 \cdot 10^{-13} \cdot W} = 6.6174
\]

\[
SIR_{n \_dB} := 10 \cdot \log(SIR_n) = 8.2068
\]

Compare this result with the target signal-to-noise ratio after integration of multiple pulses, \( SNR_{n \_dB} \) (dB), from Exercise 3-15. As shown below, the self-protection noise jammer has reduced the S/N by about 9.9 dB (almost an order of magnitude, power of 10).

\[
SNR_{n \_dB} = 18.1152
\]

\[
SNR_{n \_dB} - SIR_{n \_dB} = 9.9084
\]

\[
= 18.1152 - 8.2068 = 9.9084
\]
**Exercise 10-3** Compute the burnthrough range, $R_{bt}$ (meters), and noise jamming-to-signal ratio after integration of multiple pulses, $(J_N/S)_n$ (no units and dB) for the self-protection noise jammer from Exercise 10-2. Compare the burnthrough range to the detection range from Exercise 3-15. [Hint: use the results from Exercises 2-5 and 3-15]

We start by defining the values for all the terms and necessary constants. Convert from dB to absolute as needed.

- $P_J \equiv 25 \cdot W$
- $G_{JR\_dBi} \equiv 3$
- $G_{JR} := 10 \left( \frac{G_{JR\_dBi}}{10} \right) = 1.9953$
- $L_{J\_dB} \equiv 15$
- $L_{J} := 10 \left( \frac{L_{J\_dB}}{10} \right) = 31.6228$
- $B_J \equiv 200 \times 10^{6} \cdot Hz$
- $B_J = 750 \times 10^{3} \cdot Hz$
- $P_R \equiv 800 \times 10^{3} \cdot W$
- $G_{RT\_dBi} \equiv 38$
- $G_{RT} := 10 \left( \frac{G_{RT\_dBi}}{10} \right) = 6.3096 \times 10^{3}$
- $f_c \equiv 3 \times 10^{9} \cdot Hz$
- $G_{sp} \equiv 1$
- $L_{R\_dB} \equiv 13$
- $L_{R} := 10 \left( \frac{L_{R\_dB}}{10} \right) = 19.9526$
- $B_R \equiv 750 \times 10^{3} \cdot Hz$
- $SNR_{dt\_dB} \equiv 12$
- $SNR_{dt} := 10 \left( \frac{SNR_{dt\_dB}}{10} \right) = 15.8489$
- $\sigma \equiv 5 \cdot m^{2}$
- $R_{RT} \equiv 150 \times 10^{3} \cdot m$
- $R_{RT} := 10 \left( \frac{R_{RT\_dB}}{10} \right) = 102.2893 \cdot km$
- $G_{I} \equiv 9.745$
- $P_{J} \cdot G_{RT} \cdot \sigma \cdot G_{sp} \cdot G_{I} \cdot \left( \frac{B_{J}}{B_{R}} \right) \cdot \frac{L_{J}}{P_{J} \cdot G_{JR}} = 102.2893 \cdot km$

We can compute the burnthrough range, $R_{bt}$ (meters), of the radar using Equation 10-19.
The detection range from Exercise 3-15 is about 213 km. Wow, a little jammer goes a long way toward reducing the detection range.

We can use the logarithmic form of the burnthrough range equation as well, Equation 10-20. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the burnthrough range.

\[
\text{term}_1 := 10 \log \left( \frac{P_R}{1 \text{W}} \right) + G_{RT \_dBi} + 10 \log \left( \frac{\sigma}{1 \text{m}^2} \right) + 10 \log (G_{sp}) = 104.0206
\]

\[
= 10 \cdot \log \left( \frac{800 \times 10^3 \text{W}}{(1 \cdot \text{W})} \right) + 38 + 10 \cdot \log \left( \frac{5 \text{m}^2}{(1 \cdot \text{m}^2)} \right) + 10 \cdot \log (1) = 104.0206
\]

\[
\text{term}_2 := 10 \log (G_I) + 10 \log \left( \frac{B_J}{1 \text{Hz}} \right) + L_{J\_dB} - 10 \log (4 \pi) = 96.906
\]

\[
= 10 \cdot \log (9.745) + 10 \cdot \log \left( \frac{200 \times 10^6 \text{Hz}}{(1 \cdot \text{Hz})} \right) + L_{J\_dB} - 10 \cdot \log (4 \cdot \pi) = 96.906
\]

\[
\text{term}_3 := -\text{SNR}_{dt\_db} - L_{R\_db} - 10 \log \left( \frac{B_R}{1 \text{Hz}} \right) - 10 \log \left( \frac{P_J}{1 \text{W}} \right) - G_{JR \_dBi} = -100.73
\]

\[
= -12 - 13 - 10 \log \left( \frac{750 \times 10^3 \text{Hz}}{(1 \cdot \text{Hz})} \right) - 10 \log \left( \frac{25 \text{W}}{(1 \cdot \text{W})} \right) - 3 = -100.7
\]

\[
\left( \text{term}_1 + \text{term}_2 + \text{term}_3 \right) \cdot m = 1.0229 \times 10^5 m
\]

\[
10 \left( \frac{104.0206 + 96.906 - 100.73}{20} \right) \cdot m = 1.0229 \times 10^5 m
\]
We can compute the self-protection jammer noise jamming-to-signal ratio after integration of multiple pulses, $JN2S_n$ (no units) and $JN2S_n\_dB$ (dB), using Equation 10-21:

$$JN2S_n := \frac{P_J \cdot G_{JR}}{L_J} \cdot \left( \frac{B_R}{B_J} \right) \cdot \left( \frac{4\pi \cdot R_{RT}^2 \cdot L_R}{P_R \cdot G_{RT} \cdot \sigma \cdot G_{sp} \cdot G_I} \right) = 0.1357$$

$$\left( \frac{25\,W}{10} \right) \cdot \left( \frac{10}{10} \right) \cdot \left( \frac{750 \times 10^3 \cdot \text{Hz}}{200 \times 10^6 \cdot \text{Hz}} \right) \cdot \left( \frac{4\pi}{10} \right) \cdot \left( \frac{150 \times 10^3 \cdot \text{m}^2}{10} \right) \cdot \left( \frac{13}{10} \right) = 0.1357$$

$$JN2S_n\_dB := 10 \cdot \log(JN2S_n) = -8.6748 = 10 \cdot \log(0.1357) = -8.6742$$

As we can see from above, the $(JN/S)_n$ is negative (dB), telling use the jammer noise power is less than the target signal power. As stated in Section 10.3.2, often times people think a negative $(JN/S)_n$ means the jammer is ineffective. When as we showed above, the noise jammer is actually very effective in reducing the detection range of the radar system. This is an numerical example of why $(JN/S)_n$ is a poor metric for noise jammers.

We can use the logarithmic form of the $(JN/S)_n$ equation as well, Equation 10-22. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the $(JN/S)_n$.

$$\text{term}_4 := 10 \log \left( \frac{P_J}{1\,W} \right) + G_{JR\_dBi} + 10 \log \left( \frac{B_R}{1\,\text{Hz}} \right) + 10 \log \left( 4\pi \right) = 86.7221$$

$$= 10 \cdot \log \left[ \frac{25\,W}{(1\,W)} \right] + 3 + 10 \cdot \log \left[ \frac{750 \times 10^3 \cdot \text{Hz}}{(1\,\text{Hz})} \right] + 10 \cdot \log \left( 4 \cdot \pi \right) = 86.72$$

$$\text{term}_5 := 20 \log \left( \frac{R_{RT}}{1\,\text{m}} \right) + L_{R\_dBi} - L_{J\_dBi} - 10 \log \left( \frac{B_J}{1\,\text{Hz}} \right) = 18.5115$$
\[
\begin{align*}
\text{term}_6 & := -10 \log \left( \frac{P_R}{1 \text{W}} \right) - G_{RT\_dB} - 10 \log \left( \frac{\sigma}{1 \text{m}^2} \right) - 10 \log (G_{SP}) = -104.0206 \\
-10 \log \left( \frac{800 \times 10^3 \cdot \text{W}}{1 \text{W}} \right) - 38 - 10 \log \left( \frac{5 \cdot \text{m}^2}{1 \text{m}^2} \right) - 10 \log (1) & = -104.0206 \\
\text{term}_7 & := -10 \log (G_I) = -9.8878 \quad = (-10) \cdot \log (9.745) = -9.8878 \\
10 \left( \frac{\text{term}_4 + \text{term}_5 + \text{term}_6 + \text{term}_7}{10} \right) & = 0.1357 \\
10 \left( \frac{86.7221 + 18.5115 - 104.0206 - 9.8878}{10} \right) & = 0.1357 \\
\text{term}_4 + \text{term}_5 + \text{term}_6 + \text{term}_7 & = -8.6748 \\
86.7221 + 18.5115 - 104.0206 - 9.8878 & = -8.6748
\end{align*}
\]
**Exercise 10-4**  The self-protection jammer from Exercise 10-1 uses a false target waveform perfectly matched to the radar waveform. Compute the false target jamming-to-noise ratio after integration of multiple pulses, \((J/N)_n\) (no units and dB). Compare the false target signal-to-noise ratio with the target signal-to-noise ratio after integration of multiple pulses from Exercise 3-15. [Hint: use the results from Exercises 2-5 and 3-15]

We start by defining the values for all the terms and necessary constants. Also, include the results from Exercises 2-5, 3-15, and 10-1.

\[
\begin{align*}
J & = 2.8012 \times 10^{-11} \text{W} \\
N & = 1.1949 \times 10^{-14} \text{W} \\
G_{sp} & = 1 \\
G_I & = 9.745
\end{align*}
\]

We can compute the false target jamming-to-noise ratio after integration of multiple pulses, \(J_{NR_n}\) (no units) and \(J_{NR_n}_{dB}\) (dB), using Equation 10-24.

\[
J_{NR_n} := \frac{J \cdot G_{sp} \cdot G_I}{N} = 2.2845 \times 10^4
\]

\[
J_{NR_n}_{dB} := 10 \log(J_{NR_n}) = 43.5879
\]

Compare this result with the target signal-to-noise ratio after integration of multiple pulses, \(SNR_{n_{dB}}\) (dB), from Exercise 3-15. As shown below, the false target jammer-to-noise ratio is about 25.5 dB higher (353 times) than the target signal-to-noise ratio. This bodes well for the jammer, and poorly for the radar system.

\[
SNR_{n_{dB}} = 18.1152
\]

\[
J_{NR_n}_{dB} - SNR_{n_{dB}} = 25.4727
\]

\[
= 43.5879 - 18.1152 = 25.4727
\]
**Exercise 10-5** Compute the false target jamming-to-signal ratio after integration of multiple pulses, (J/S)_n (no units and dB), and the false target radar detection range, R_{dt} (meters), for the self-protection false target jammer from Exercise 10-4. Compare the false target radar detection range to the radar detection range for the true target in Exercise 3-15. [Hint: use the results from Exercises 2-5 and 3-15]

We start by defining the necessary results from Exercise 10-4. We then define the values for all the terms and necessary constants. Convert from dB to absolute as needed.

\[
J = 2.8012 \times 10^{-11} \cdot W \\
S = 7.9445 \times 10^{-14} \cdot W \\
N = 1.1949 \times 10^{-14} \cdot W \\
G_I = 9.745 \\
P_J = 25 \cdot W \\
G_{JR\_dbi} = 3 \\
L_{J\_db} = 15 \\
G_{RT\_dbi} = 38 \\
f_c = 3 \times 10^9 \cdot Hz \\
G_{SP} = 1 \\
L_{R\_db} = 13 \\
\text{SNR}_{dt\_db} = 12 \\
c = 3 \times 10^8 \cdot \frac{m}{sec}
\]

\[
G_{JR} := 10^{\frac{3}{10}} = 1.9953 \\
L_J := 10^{\frac{15}{10}} = 31.6228 \\
G_{RT} := 10^{\frac{38}{10}} = 6.3096 \times 10^3 \\
\text{SNR}_{dt} := 10^{\frac{12}{10}} = 15.8489 \\
L_R := 10^{\frac{13}{10}} = 19.9526
\]

We compute the false target jamming-to-target signal ratio after integration of multiple pulses, J2S_n (no units and J2S_n dB), using Equation 10-25.
\[ J_{2S_n} := \frac{J}{S} = 352.5961 \]
\[ J_{2S_n\_dB} := 10 \log(J_{2S_n}) = 25.4728 \]

We compute the false target radar detection range, \( R_{dt} \) (meters), using Equation 10-27. We need the wavelength, \( \lambda \) (meters), for this equation and compute it using Equation 2-2.

\[ \lambda := \frac{c}{f_c} = 0.1 \text{ m} \]
\[ \frac{3 \cdot 10^8 \left( \frac{\text{m}}{\text{sec}} \right)}{3 \cdot 10^9 \cdot \text{Hz}} = 0.1 \text{ m} \]
\[ R_{dt} := \sqrt{\frac{P_J \cdot G_{JR} \cdot G_{RT} \cdot \lambda^2 \cdot G_s \cdot G_I}{(4 \pi)^2 \cdot \text{SNR}_{dt} \cdot N \cdot L_J}} = 5.6949 \times 10^6 \text{ m} \]
\[ R_{dt} = 5.6949 \times 10^3 \cdot \text{km} \]

Compare to the target radar detection range, \( R_{dt\_tgt} \) (meters), from Exercise 3-15. Wow! A little jammer goes a long way in providing an extremely long detection range.

\[ R_{dt\_tgt} = 213.3 \cdot \text{km} \]
\[ \frac{R_{dt}}{R_{dt\_tgt}} = 26.699 \]
\[ \frac{5.6949 \times 10^6 \text{ m}}{(213.3 \cdot \text{km})} = 26.699 \]
We can obtain an understanding for why the false target jammer detection range is significantly greater than the target radar detection range by comparing the jammer effective radiated power (ERP), from Exercise 10-1, to the reflected power from the target, from Exercise 2-5. The jammer ERP is about 31.5 W, while the reflected power from the target is about 0.06 W. The significantly higher jammer ERP contributes to a significantly longer detection range.
**Exercise 10-6** To determine if the noise jamming power will be above the thermal noise in the receiver, calculate the ratio of jamming noise to receiver thermal noise out of the radar receiver. The target/jammer is at a range \( R_{RT} = 200 \) km, power \( P_J = 10 \) Watts, antenna gain in the direction of the radar \( G_{JR} = 1 \) dBi, noise bandwidth \( B_J = 50 \times 10^6 \) Hz, and jammer-related losses \( L_J = 3 \) dB. The radar has an antenna gain in the direction of the target/jammer \( G_{RT} = 40 \) dBi, receiver noise figure \( F_R = 7 \) dB, wavelength \( \lambda = 0.05 \) meters, and the receiver filter bandwidth matched to a 1 \( \mu \)sec pulse.

We start by defining the values for all the terms and necessary constants. Convert from dB to absolute as needed.

\[
R_{RT} = 200 \times 10^3 \text{ m} \quad P_J = 10 \cdot \text{W} \quad G_{JR\text{ dBi}} = 1 \quad B_J = 50 \times 10^6 \cdot \text{Hz} \quad L_J\text{ dBi} = 3 \quad G_{RT\text{ dBi}} = 40
\]

\[
F_R\text{ dB} = 7 \quad \lambda = 0.05 \cdot \text{m} \quad \tau = 1 \times 10^{-6} \cdot \text{sec} \quad T_0 = 290 \cdot \text{K} \quad k = 1.38 \times 10^{-23} \cdot \frac{\text{W} \cdot \text{sec}}{\text{K}}
\]

\[
G_{JR} := 10\left(\frac{G_{JR\text{ dBi}}}{10}\right) = 1.2589 \\
L_J := 10\left(\frac{L_J\text{ dBi}}{10}\right) = 1.9953 \\
G_{RT} := 10\left(\frac{G_{RT\text{ dBi}}}{10}\right) = 1 \times 10^4
\]

\[
F_R := 10\left(\frac{F_R\text{ dB}}{10}\right) = 5.0119
\]

We will start by computing the jamming noise power out of the radar receiver, \( J_N \) (W), using Equation 10-13. We were given the vast majority of the terms needed for this equation. The only value we are missing is the radar receiver bandwidth. The radar uses a matched filter bandwidth, thus the radar receiver bandwidth computed using Equation 2-15.

\[
J_N = \frac{P_J \cdot G_{JR} \cdot G_{RT} \cdot \lambda^2}{(4\pi)^2 \cdot R_{RT}^2 \cdot L_J} \cdot \left(\frac{B_R}{B_J}\right)
\]

\[
B_R := \frac{1}{\tau} = 1 \times 10^6 \cdot \text{Hz}
\]

\[
= \frac{1}{\left(1 \times 10^{-6} \cdot \text{sec}\right)} = 1 \times 10^6 \cdot \text{Hz}
\]
We compute the radar receiver thermal noise power, \( N \) (Watts), using Equation 2-14.

\[
N := F_R \cdot k \cdot T_0 \cdot B_R = 2.0058 \times 10^{-14} \text{W}
\]

\[
\frac{J_N}{N} = 24.9008
\]

We compute the ratio of the jamming noise to the receiver thermal noise, \( \frac{J_N}{N} \) (no units and dB), as shown below. This computation shows us the jamming noise power is 24.9 times higher in magnitude to the receiver thermal noise power.

\[
10 \log \left( \frac{J_N}{N} \right) = 13.9621
\]

We can use this information to confirm it is numerically correct to use Equation 10-18, \( (S/J_N)_h \), and Equation 10-19, burnthrough range. Both these equations assume \( J_N \gg N \), which is certainly the case for this exercise.
**Exercise 10-7**  The jammer in the preceding exercise (10-6) is aboard an aircraft flying toward the radar which has a peak power $P_R = 1$ MW, radar-related losses $L_R = 6$ dB, signal processing gain $G_{sp} = 1$, and integration gain $G_I = 15$ dB. What is the target signal-to-jamming ratio after integration of multiple pulses, $(S/J_N)_n$ (no units and dB), if the aircraft has a nose-on radar cross section $\sigma = 1$ m$^2$, of 10 m$^2$?

Collect the jammer and radar terms from Exercise 10-6. Then we define the values for all the terms and constants, and convert from decibels to absolute as needed.

$$
\begin{align*}
P_R &= 1 \times 10^6 \cdot W \\
L_{R\_dB} &= 6 \\
G_{sp} &= 1 \\
G_{I\_dB} &= 15 \\
\sigma_1 &= 1 \cdot m^2 \\
\sigma_2 &= 10 \cdot m^2 \\
R_{RT} &= 200 \times 10^{-3} \cdot m \\
P_J &= 10 \cdot W \\
G_{JR\_dBi} &= 1 \\
B_J &= 50 \times 10^6 \cdot Hz \\
L_{J\_dB} &= 3 \\
G_{RT\_dBi} &= 40 \\
\tau &= 1 \times 10^{-6} \cdot sec \\
L_R &= 10^{\left(\frac{L_{R\_dB}}{10}\right)} = 3.9811 \\
G_I &= 10^{\left(\frac{G_{I\_dB}}{10}\right)} = 31.6228 \\
G_{JR} &= 10^{\left(\frac{G_{JR\_dBi}}{10}\right)} = 1.2589 \\
L_J &= 10^{\left(\frac{L_{J\_dB}}{10}\right)} = 1.9953 \\
G_{RT} &= 10^{\left(\frac{G_{RT\_dBi}}{10}\right)} = 1 \times 10^4 \\
\end{align*}
$$

We will compute the target signal-to-jamming ratio after integration of multiple pulses, $(S/J_N)_n$ (no units), using Equation 10-18. We can use this equation because of the result of Exercise 10-6. We were given the vast majority of the terms needed for this equation. The only value we are missing is the radar receiver bandwidth. The radar uses a matched filter bandwidth, thus the radar receiver bandwidth is given by Equation 2-15.
We now compute the target signal-to-jamming ratio for the first aircraft RCS, \( \text{SJR}_{n1} \) (no units and dB), and then for the second aircraft RCS, \( \text{SJR}_{n2} \) (no units and dB).

\[
\text{SJR}_n = \frac{P_R \cdot G_{RT} \cdot \sigma \cdot G_{sp} \cdot G_I}{4\pi \cdot R_{RT}^2 \cdot L_R} \cdot \left( \frac{B_j}{B} \right) \cdot \frac{L_J}{P_J \cdot G_{JR}}
\]

\[B_R := \frac{1}{\tau} = 1 \times 10^6 \cdot \text{Hz} \]

\[
= \frac{1}{(1 \cdot 10^{-6} \cdot \text{sec})} = 1 \times 10^6 \cdot \text{Hz}
\]

\[
\text{SJR}_{n1} := \frac{P_R \cdot G_{RT} \cdot \sigma_1 \cdot G_{sp} \cdot G_I}{4\pi \cdot R_{RT}^2 \cdot L_R} \cdot \left( \frac{B_j}{B_R} \right) \cdot \frac{L_J}{P_J \cdot G_{JR}} = 1.2523
\]

\[
\left( \frac{1 \times 10^6 \cdot \text{W}}{4\pi \cdot (200 \times 10^3 \cdot \text{m})^2} \right) \cdot \left[ \frac{\left( \frac{40}{10} \right)}{10} \right] \cdot (1 \cdot \text{m}^2) \cdot (1) \cdot \left[ \frac{\left( \frac{15}{10} \right)}{10} \right] \cdot \left( \frac{50 \times 10^6 \cdot \text{Hz}}{1 \times 10^6 \cdot \text{Hz}} \right) \cdot \frac{3}{10} = 1.2523
\]

\[
10 \log (\text{SJR}_{n1}) = 0.977
\]

\[
= 10 \cdot \log (1.2523) = 0.9771
\]

\[
\text{SJR}_{n2} := \frac{P_R \cdot G_{RT} \cdot \sigma_2 \cdot G_{sp} \cdot G_I}{4\pi \cdot R_{RT}^2 \cdot L_R} \cdot \left( \frac{B_j}{B_R} \right) \cdot \frac{L_J}{P_J \cdot G_{JR}} = 12.5228
\]

\[
\left( \frac{1 \times 10^6 \cdot \text{W}}{4\pi \cdot (200 \times 10^3 \cdot \text{m})^2} \right) \cdot \left[ \frac{\left( \frac{40}{10} \right)}{10} \right] \cdot (10 \cdot \text{m}^2) \cdot (1) \cdot \left[ \frac{\left( \frac{15}{10} \right)}{10} \right] \cdot \left( \frac{50 \times 10^6 \cdot \text{Hz}}{1 \times 10^6 \cdot \text{Hz}} \right) \cdot \frac{3}{10} = 12.5228
\]

\[
10 \log (\text{SJR}_{n2}) = 10.977
\]

\[
= 10 \cdot \log (12.5228) = 10.977
\]
**Exercise 10-8** Assuming a signal-to-jamming ratio $S/J = 6$ dB is required for detection. Using the radar and jammer characteristics from the previous two exercises (10-6 and 10-7), what is the burnthrough range of the radar for a target with a radar cross section $\sigma = 1 \text{ m}^2$, of $10 \text{ m}^2$?

Collect the jammer and radar terms from Exercises 10-6 and 10-7. Define the values for all the terms and constants, and convert from decibels to absolute as needed.

\[
\begin{align*}
\text{SJR}_{\text{dB}} & \equiv 6 & \sigma_1 & \equiv 1 \cdot \text{m}^2 & \sigma_2 & \equiv 10 \cdot \text{m}^2 & P_R & \equiv 1 \times 10^6 \cdot \text{W} & L_{R_{\text{dB}}} & \equiv 6 & G_{\text{sp}} & \equiv 1 \\
G_{I_{\text{dB}}} & \equiv 15 & R_{RT} & \equiv 200 \times 10^3 \cdot \text{m} & P_J & \equiv 10 \cdot \text{W} & G_{JR_{\text{dB}}} & \equiv 1 & B_J & \equiv 50 \times 10^6 \cdot \text{Hz} & L_{J_{\text{dB}}} & \equiv 3 \\
G_{RT_{\text{dB}}} & \equiv 40 & \tau & \equiv 1 \times 10^{-6} \cdot \text{sec} & \text{SJR} & \equiv 10^{\left( \frac{\text{SJR}_{\text{dB}}}{10} \right)} = 3.9811 & \text{L}_R & \equiv 10^{\left( \frac{L_{R_{\text{dB}}}}{10} \right)} = 3.9811 & \text{G}_I & \equiv 10^{\left( \frac{G_{I_{\text{dB}}}}{10} \right)} = 31.6228 \\
\text{G}_{JR} & \equiv 10^{\left( \frac{G_{JR_{\text{dB}}}}{10} \right)} = 1.2589 & \text{L}_J & \equiv 10^{\left( \frac{L_{J_{\text{dB}}}}{10} \right)} = 1.9953 & \text{G}_{RT} & \equiv 10^{\left( \frac{G_{RT_{\text{dB}}}}{10} \right)} = 1 \times 10^4
\end{align*}
\]

The burnthrough range, $R_{bt}$ (meters), is given by Equation 10-19. We can use this equation because of the result of Exercise 10-6. We were given the vast majority of the terms needed for this equation. We are missing the radar detection threshold and radar receiver bandwidth. The radar detection threshold is the same as the signal-to-jamming ratio required for detection. The radar uses a matched filter bandwidth, thus the radar receiver bandwidth is given by Equation 2-15.
We now compute the burnthrough range for the first aircraft RCS, \( R_{bt1} \) (m), and then for the second aircraft RCS, \( R_{bt2} \) (m).

\[
R_{bt1} = \sqrt{\frac{P_R \cdot G_{RT} \cdot \sigma_1 \cdot G_{sp} \cdot G_I \cdot \left( \frac{B_J}{B_R} \right) \cdot \frac{L_J}{P_J \cdot G_{JR}}}{4 \cdot \pi \cdot SNR_{dt} \cdot L_R}} = 1.1217 \times 10^5 \text{m} \\
R_{bt1} = 112.1709 \cdot \text{km}
\]

\[
\frac{1 \cdot 10^{-6} \cdot \text{sec}}{1 \cdot 10^6 \cdot \text{Hz}} = 1 \times 10^6 \cdot \text{Hz}
\]

\[
R_{bt2} = \sqrt{\frac{P_R \cdot G_{RT} \cdot \sigma_2 \cdot G_{sp} \cdot G_I \cdot \left( \frac{B_J}{B_R} \right) \cdot \frac{L_J}{P_J \cdot G_{JR}}}{4 \cdot \pi \cdot SNR_{dt} \cdot L_R}} = 3.5472 \times 10^5 \text{m} \\
R_{bt2} = 354.7154 \cdot \text{km}
\]
**Exercise 10-9** A radar system attempts to track a target with a radar cross section $\sigma = 5 \text{ m}^2$ at a radar-to-target range $R_{RT} = 15 \text{ km}$. The target releases chaff passive expendable and the radar system tracks the chaff as it separates from the target. The radar mainbeam antenna gain in the direction of the chaff passive expendable is $G_{RE} = 35 \text{ dBi}$. The target is separated in angle from the chaff by one half of the antenna half-power beamwidth. The chaff is 50 meters further in range than the target. What is the passive expendable jamming-to-signal ratio after integration of multiple pulses, $(J_{PE}/S)_n$ (no units and dB)? Use the average radar cross section of the chaff from Exercise 6-5.

We start by defining the values for all the terms and necessary constants. We also define the average radar cross section of the chaff from Exercise 6-5, $\sigma_{pe}$ (square meters).

\[
\sigma = 5 \cdot \text{m}^2 \quad R_{RT} = 15 \times 10^3 \cdot \text{m} \quad G_{RE \_\text{dBi}} = 35 \quad \sigma_{pe} = 1.35 \times 10^3 \cdot \text{m}^2
\]

Our first step is to compute the radar antenna gain in the direction of the target, $G_{RT \_\text{dBi}}$ (dBi). The target is separated in angle from the chaff by one half of the antenna half-power beamwidth (see Section 4.1.1). Thus, the radar antenna gain in the direction of the target is 3 dB less than the radar antenna gain in the direction of the chaff.

\[
G_{RT \_\text{dBi}} := G_{RE \_\text{dBi}} - 3 = 32
\]

Our second step is to compute the radar-to-passive expendable range, $R_{RE}$ (meters). The chaff is 50 meters further in range than the target.

\[
R_{RE} := R_{RT} + 50 \cdot \text{m} = 1.505 \times 10^4 \cdot \text{m}
\]

Now we are ready to compute the passive expendable jamming-to-signal ratio after integration of multiple pulses, $J_{PE}/S_n$ (no units and dB), using Equation 10-33. Since we are using an algebraic equation we need to convert the antenna gain values from decibels to absolute.

\[
G_{RT} := 10 \left( \frac{G_{RT \_\text{dBi}}}{10} \right) = 1.5849 \times 10^3
\]

\[
G_{RE} := 10 \left( \frac{G_{RE \_\text{dBi}}}{10} \right) = 3.1623 \times 10^3
\]

\[
= 10 \left( \frac{32}{10} \right) = 1.5849 \times 10^3
\]

\[
= 10 \left( \frac{35}{10} \right) = 3.1623 \times 10^3
\]
\[ JPE2S_n := \frac{G_{RE}^2 \cdot \frac{\sigma_{pe}}{\sigma} \cdot R_{RT}^4}{G_{RT}^2} = 1.0607 \times 10^3 \]

\[
= \left[ \frac{10}{10} \right]^2 \cdot \left[ \frac{1.35 \cdot 10^3 \cdot \text{m}^2}{(5 \cdot \text{m}^2)} \right] \cdot \left[ \frac{(15 \cdot 10^3 \cdot \text{m})^4}{(1.505 \cdot 10^4 \cdot \text{m})^4} \right] = 1.0607 \times 10^3
\]

\[10 \log (JPE2S_n) = 30.2558\]

We can use the logarithmic form of this equation as well, Equation 10-34. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the passive expendable jamming-to-signal ratio.

\[ \text{term}_1 := 2 \times G_{RE} \text{dbi} + 10 \log \left( \frac{\sigma_{pe}}{1 \text{m}^2} \right) + 40 \log \left( \frac{R_{RT}}{1 \text{m}} \right) = 268.347 \]

\[= 2 \cdot 35 + 10 \log \left( \frac{1.35 \cdot 10^3 \cdot \text{m}^2}{(1 \cdot \text{m}^2)} \right) + 40 \log \left( \frac{15 \cdot 10^3 \cdot \text{m}}{(1 \cdot \text{m})} \right) = 268.347 \]

\[ \text{term}_2 := -2 \times G_{RT} \text{dbi} - 10 \log \left( \frac{\sigma}{1 \text{m}^2} \right) - 40 \log \left( \frac{R_{RE}}{1 \text{m}} \right) = -238.0912 \]

\[= (-2) \cdot 32 - 10 \log \left( \frac{5 \cdot \text{m}^2}{(1 \cdot \text{m}^2)} \right) - 40 \log \left( \frac{1.505 \cdot 10^4 \cdot \text{m}}{(1 \cdot \text{m})} \right) = -238.0912 \]

\[
\frac{\text{term}_1 + \text{term}_2}{10} \cdot \text{m} = 1.0607 \times 10^3 \text{ m} \]

\[
10 \left( \frac{268.347 - 238.0912}{10} \right) = 1.0607 \times 10^3 \]

\[\text{term}_1 + \text{term}_2 = 30.2558 \]

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**Exercise 11-1**  A support jammer has the following characteristics: peak transmit power $P_J = 1000$ Watts, transmit antenna gain in the direction of the radar $G_{JR} = 3$ dBi, total jammer-related losses $L_J = 15$ dB, and jammer transmit loss $L_{Jt} = 2$ dB. The jammer is 100 km, $R_{RJ}$, from the radar system from Exercises 2-5 and 3-15. The radar antenna gain in the direction of the jammer, $G_{RJ}$, is 20 dB less than the radar antenna gain in the direction of the target, $G_{RT}$. Compute the following: (a) the transmitted jammer effective radiated power, ERP$_J$ (Watts and dBW), (b) the received jammer power density at the radar receive antenna, and (c) the received jammer power, $J$ (Watts and dBW). Compare the received jammer power to the jammer transmitted effective radiated power. [Hint: use the results from Exercises 2-5 and 3-15]

We start by defining the values for all the terms and necessary constants. We also define the values for all the necessary terms from Exercises 2-5 and 3-15.

$$
\begin{align*}
P_J & = 1000\cdot W  \\
G_{JR} & = 3 \text{ dBi}  \\
L_J & = 15 \text{ dB}  \\
L_{Jt} & = 2 \text{ dB}  \\
R_{RJ} & = 100 \times 10^3 \cdot m  \\
G_{RT} & = 38 \text{ dBi}  \\
f_c & = 3 \times 10^9 \cdot \text{Hz}  \\
c & = 3 \times 10^8 \cdot \frac{m}{\text{sec}} \\
\end{align*}
$$

We first need to compute the radar antenna gain in the direction of the jammer, $G_{RJ}$ (dBi). As stated in the exercise, the radar antenna gain in the direction of the jammer is 20 dB less than the radar antenna gain in the direction of the target, $G_{RT}$ (dBi). The calculation is shown below.

$$
G_{RJ} := G_{RT} - 20 = 18
$$

Convert from dB to absolute as needed.

$$
\begin{align*}
G_{JR} & := 10 \log \left( \frac{G_{JR}}{10} \right) = 1.9953  \\
L_J & := 10 \log \left( \frac{L_J}{10} \right) = 31.6228  \\
L_{Jt} & := 10 \log \left( \frac{L_{Jt}}{10} \right) = 1.5849
\end{align*}
$$
(a) Compute the transmitted effective radiated power (ERP) of the support jammer, ERP\(_J\) (Watts) and ERP\(_J\)\(_{\text{dBW}}\) (dBW), as given in Equation 11-1.

\[
\text{ERP}_J := \frac{P_J \cdot G_{JR}}{L_{Jt}} = 1.2589 \times 10^3 \text{W}
\]

\[
\text{ERP}_J_{\text{dBW}} := 10 \log \left( \frac{\text{ERP}_J}{1\text{W}} \right) = 31
\]

(b) Compute the support jammer power density at the radar receive antenna, as given in Equation 11-2.

\[
\frac{P_J \cdot G_{JR}}{4\pi R_{RJ}^2 \cdot L_{Jt}} = 1.0018 \times 10^{-8} \cdot \frac{\text{W}}{\text{m}^2}
\]

(c) Compute the received support jammer power, J (Watts) and J\(_{\text{dBW}}\) (dBW), in the radar receiver, as given in Equation 11-5. We will need the wavelength, \(\lambda\) (meters), and will compute it using Equation 2-2.
\[ \lambda := \frac{c}{f_c} = 0.1 \text{ m} \]

\[ J := \frac{P_J \cdot G_{JR} \cdot G_{BJ} \cdot \lambda^2}{(4\pi)^2 \cdot R_{RJ}^2 \cdot L_J} = 2.521 \times 10^{-11} \text{ W} \]

\[ J_{\text{dBW}} := 10 \cdot \log \left( \frac{J}{1 \text{ W}} \right) = -105.9842 \]

\[ \frac{J}{ERP_J} = 2.0025 \times 10^{-14} \]

To compare the received support jammer power to the support jammer's transmitted ERP we compute the ratio of the two. The radar received approximately 14 orders of magnitude (power of 10) less power than the jammer transmitted. The vast majority of the power is lost by the \(1/R^2\) propagation from the jammer to the radar, here a 100 km propagation path length.
**Exercise 11-2**  The support jammer from Exercise 11-1 uses a noise waveform with a bandwidth $B_J = 200$ MHz centered about the radar carrier frequency. Compute the following: (a) the jammer noise power in the radar receiver, $J_N$ (Watts and dBW), (b) the interference signal power, $I$ (Watts and dBW), and (c) the target signal-to-interference ratio after integration of multiple pulses, $(S/I)_n$ (no units and dB). Compare the target signal-to-interference ratio with the target signal-to-noise ratio after integration of multiple pulses from Exercise 3-15. [Hint: use the results of Exercises 2-5 and 3-15]

We start by defining the values for all the terms and necessary constants.

$$B_J \equiv 200 \times 10^6 \cdot \text{Hz} \quad B_R \equiv 750 \times 10^3 \cdot \text{Hz} \quad G_{sp} \equiv 1 \quad G_I \equiv 9.745$$

$$S \equiv 7.9445 \times 10^{-14} \cdot \text{W} \quad N \equiv 1.1949 \times 10^{-14} \cdot \text{W} \quad J \equiv 2.521 \times 10^{-11} \cdot \text{W}$$

(a) Compute the support jammer noise power in the radar receiver, $J_N$ (Watts) and $J_{N\_\text{dBW}}$ (dBW), as given in Equation 11-13.

$$J_N := J \cdot \left( \frac{B_R}{B_J} \right) = 9.4537 \times 10^{-14} \cdot \text{W}$$

$$= \frac{750 \times 10^3 \cdot \text{Hz}}{200 \times 10^6 \cdot \text{Hz}} = 9.4537 \times 10^{-14} \cdot \text{W}$$

$$J_{N\_\text{dBW}} := 10 \cdot \log \left( \frac{J_N}{1 \cdot \text{W}} \right) = -130.244$$

(b) Compute the interference signal power, $I$ (Watts) and $I_{\_\text{dBW}}$ (dBW), as given in Equation 11-14.

$$I := N + J_N = 1.0649 \times 10^{-13} \cdot \text{W}$$

$$= 1.1949 \times 10^{-14} \cdot \text{W} + 9.4537 \times 10^{-14} \cdot \text{W} = 1.0649 \times 10^{-13} \cdot \text{W}$$

$$I_{\_\text{dBW}} := 10 \cdot \log \left( \frac{I}{1 \cdot \text{W}} \right) = -129.7271$$

(c) Compute the multiple pulse signal-to-interference ratio, $SIR_n$ (no units) and $SIR_{n\_\text{dB}}$ (dB), after non-coherent integration, as given in Equation 11-17.

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\[
SIR_n := \frac{S \cdot G_{sp} \cdot G_l}{N + J_N} = 7.2703
\]

\[
SIR_{n\,\text{dB}} := 10 \cdot \log (SIR_n) = 8.6155
\]

Compare this result with the target signal-to-noise ratio after integration of multiple pulses, \(SNR_{n\,\text{dB}}\) (dB), from Exercise 3-15. As shown below, the support noise jammer has reduced the S/N by about 9.5 dB (almost an order of magnitude, power of 10).

\[
SNR_{n\,\text{dB}} = 18.1152
\]

\[
SNR_{n\,\text{dB}} - SIR_{n\,\text{dB}} = 9.4997
\]
Exercise 11-3  Compute the burnthrough range, \( R_{bi} \) (meters), and noise jamming-to-signal ratio after integration of multiple pulses, \((J/N)/S)_n\) (no units and dB) for the support noise jammer from Exercise 11-2. [Hint: use the results of Exercises 2-5 and 3-15]

We start by defining the values for all the terms and necessary constants. Convert from dB to absolute as needed.

\[ P_J \equiv 1000 \cdot W \quad G_{JR\_dBi} \equiv 3 \quad L_{J\_dB} \equiv 15 \quad B_J \equiv 200 \times 10^6 \cdot Hz \quad R_{RJ} \equiv 100 \times 10^3 \cdot m \quad P_R \equiv 800 \times 10^3 \cdot W \]
\[ G_{RT\_dBi} \equiv 38 \quad f_c \equiv 3 \times 10^9 \cdot Hz \quad G_{sp} \equiv 1 \quad L_{R\_dB} \equiv 13 \quad B_R \equiv 750 \times 10^3 \cdot Hz \quad SNR_{dt\_dB} \equiv 12 \]
\[ \sigma \equiv 5 \cdot m^2 \quad R_{RT} \equiv 150 \times 10^3 \cdot m \quad G_I \equiv 9.745 \quad c \equiv 3 \times 10^8, \frac{m}{sec} \]

We will use the values we calculated in Exercises 11-1 and 11-2.

\[ S \equiv 7.9445 \times 10^{-14} \cdot W \quad N \equiv 1.1949 \times 10^{-14} \cdot W \quad J_N \equiv 9.4537 \times 10^{-14} \cdot W \]
\[ G_{RJ\_dBi} := G_{RT\_dBi} - 20 = 18 \]

Convert from dB to absolute as needed.

\[ G_{JR} := 10 \left( \frac{G_{JR\_dBi}}{10} \right) = 1.9953 \]
\[ G_{JR} := 10 \left( \frac{3}{10} \right) = 1.9953 \]
\[ L_J := 10 \left( \frac{L_{J\_dB}}{10} \right) = 31.6228 \]
\[ L_J := 10 \left( \frac{15}{10} \right) = 31.6228 \]
\[ SNR_{dt} := 10 \left( \frac{SNR_{dt\_dB}}{10} \right) = 15.8489 \]
\[ SNR_{dt} := 10 \left( \frac{12}{10} \right) = 15.8489 \]
\[ G_{RJ} := 10 \left( \frac{G_{RJ\_dBi}}{10} \right) = 63.0957 \]
\[ G_{RJ} := 10 \left( \frac{38}{10} \right) = 63.0957 \]
We can compute the burnthrough range, $R_{bt}$ (meters), of the radar using Equation 11-19. We will need the wavelength, $\lambda$ (meters), and will compute it using Equation 2-2.

$$\lambda := \frac{c}{f_c} = 0.1\text{ m}$$

$$\lambda = \frac{3 \times 10^8 \text{ m}}{(3 \times 10^9 \text{ Hz})} = 0.1\text{ m}$$

$$R_{bt} := \frac{\frac{P_R \cdot G_{RT}^2 \cdot \lambda^2 \cdot \sigma \cdot G_{SP} \cdot G_I}{(4\pi)^3 \cdot SNR_{dt} \cdot L_R \cdot (N + J_N)}}{\sqrt{(4\pi)^3 \cdot SNR_{dt} \cdot L_R \cdot (N + J_N)}} = 1.2345 \times 10^5 \text{ m}$$

$$R_{bt} = 123.4468 \text{ km}$$

The detection range from Exercise 3-15 is about 213 km. Wow, a little jammer goes a long way toward reducing the detection range.

We can compute the support jammer noise jamming-to-signal ratio after integration of multiple pulses, $JN2S_n$ (no units) and $JN2S_n_{dB}$ (dB), using Equation 11-19.

$$JN2S_n := \frac{P_J \cdot G_{JR} \cdot G_{RJ}}{R_{RJ}^2 \cdot L_J} \cdot \left( \frac{B_R}{B_J} \right) \cdot \frac{(4\pi) \cdot R_{RT}^4 \cdot L_R}{P_R \cdot G_{RT}^2 \cdot \sigma \cdot G_{SP} \cdot G_I} = 0.1221$$
Since we have all the necessary individual terms we can also compute the support jammer noise jamming-to-signal ratio as shown below.

\[
\frac{J_N}{S \cdot G_{sp} \cdot G_I} = 0.1221
\]

\[
10 \log \left( \frac{J_N}{S \cdot G_{sp} \cdot G_I} \right) = -9.1325
\]

As we can see from above, the \((J_N/S)_h\) is negative (dB), telling use the jammer noise power is less than the target signal power. As stated in Section 11.3.2, often times people think a negative \((J_N/S)_h\) means the jammer is ineffective. When as we showed above, the noise jammer is actually very effective in reducing the detection range of the radar system. This is an numerical example of why \((J_N/S)_h\) is a poor metric for noise jammers.

We can use the logarithmic form of the \((J_N/S)_h\) equation as well, Equation 11-20. Because this equation will not fit across the page we need to divide it into multiple terms. We then put these terms together and compute the \((J_N/S)_h\).

\[
\text{term}_1 := 10 \log \left( \frac{P_J}{1 \text{W}} \right) + G_{JR\_dBi} + G_{RJ\_dBi} + 10 \log \left( \frac{B_R}{1 \text{Hz}} \right) = 109.7506
\]

\[
= 10 \cdot \log \left( \frac{1000 \cdot \text{W}}{(1 \cdot \text{W})} \right) + 3 + 18 + 10 \cdot \log \left( \frac{750 \cdot 10^3 \cdot \text{Hz}}{(1 \cdot \text{Hz})} \right) = 109.7506
\]
\[
\text{term}_2 := 10 \log (4\pi) + 40 \log \left( \frac{R_{RT}}{1\text{m}} \right) + L_{R_{dB}} - 20 \log \left( \frac{R_{RJ}}{1\text{m}} \right) = 131.0357
\]

\[
10 \log (4\pi) + 40 \log \left( \frac{150 \times 10^3 \cdot \text{m}}{1\text{m}} \right) + 13 - 20 \log \left( \frac{100 \times 10^3 \cdot \text{m}}{1\text{m}} \right) = 131.0357
\]

\[
\text{term}_3 := -L_{J_{dB}} - 10 \log \left( \frac{B_J}{1\text{Hz}} \right) - 10 \log \left( \frac{P_R}{1\text{W}} \right) - 2 \times G_{RT_{dB}} = -233.0412
\]

\[
= -15 - 10 \cdot \log \left( \frac{200 \cdot 10^6 \cdot \text{Hz}}{(1\cdot\text{Hz})} \right) - 10 \cdot \log \left( \frac{800 \cdot 10^3 \cdot \text{W}}{(1\cdot\text{W})} \right) - 2.38 = -233.0412
\]

\[
\text{term}_4 := -10 \log \left( \frac{\sigma}{1\text{m}^2} \right) - 10 \log (G_{sp}) - 10 \log (G_I) = -16.8775
\]

\[
= (-10) \cdot \log \left( \frac{5\cdot\text{m}^2}{(1\cdot\text{m}^2)} \right) - 10 \cdot \log (1) - 10 \cdot \log (9.745) = -16.8775
\]

\[
10 \left( \frac{\text{term}_1 + \text{term}_2 + \text{term}_3 + \text{term}_4}{10} \right) = 0.1221
\]

\[
\frac{\left( \frac{109.7506 + 131.0357 - 233.0412 - 16.8775}{10} \right)}{10} = 0.1221
\]

\[
\text{term}_1 + \text{term}_2 + \text{term}_3 + \text{term}_4 = -9.1324
\]

\[
\]
**Exercise 11-4** The support jammer from Exercise 11-1 uses a false target waveform perfectly matched to the radar waveform. Compute the false target jamming-to-noise ratio after integration of multiple pulses, \((J/N)_n\) (no units and dB). Compare the false target signal-to-noise ratio with the target signal-to-noise ratio after integration of multiple pulses from Exercise 3-15. [Hint: use the results from Exercises 2-5 and 3-15]

We start by defining the values for all the terms and necessary constants. Also, include the results from Exercises 2-5, 3-15, and 11-1.

\[
J \equiv 2.521 \times 10^{-11} \text{ W} \\
N \equiv 1.1949 \times 10^{-14} \text{ W} \\
G_{sp} \equiv 1 \\
G_I \equiv 9.745
\]

We can compute the false target jamming-to-noise ratio after integration of multiple pulses, \(JNR_n\) (no units) and \(JNR_{n \text{ dB}}\) (dB), using Equation 11-22.

\[
JNR_n := \frac{J \cdot G_{sp} \cdot G_I}{N} = 2.056 \times 10^4
\]

\[
JNR_{n \text{ dB}} := 10 \cdot \log(JNR_n) = 43.1302
\]

Compare this result with the target signal-to-noise ratio after integration of multiple pulses, \(SNR_{n \text{ dB}}\) (dB), from Exercise 3-15. As shown below, the false target jammer-to-noise ratio is about 25 dB higher (316 times) than the target signal-to-noise ratio. This bodes well for the jammer, and poorly for the radar system.

\[
SNR_{n \text{ dB}} \equiv 18.1152 \\
JNR_{n \text{ dB}} - SNR_{n \text{ dB}} = 25.015 = 43.1302 - 18.1152 = 25.015
\]
Exercise 11-5  Compute the false target jamming-to-signal ratio after integration of multiple pulses, \((J/S)_n\) (no units and dB), and the false target radar detection range, \(R_{dt}\) (meters), for the support false target jammer from Exercise 11-4. Compare the false target radar detection range to the radar detection range for the true target in Exercise 3-15.  [Hint: use the results from Exercises 2-5 and 3-15]

We start by defining the necessary results from Exercise 10-4. We then define the values for all the terms and necessary constants. Convert from dB to absolute as needed.

\[
\begin{align*}
J &\equiv 2.521 \times 10^{-11} \cdot W \\
S &\equiv 7.9445 \times 10^{-14} \cdot W \\
N &\equiv 1.1949 \times 10^{-14} \cdot W \\
G_I &\equiv 9.745 \\
P_J &\equiv 1000 \cdot W \\
G_{JR_dBi} &\equiv 3 \\
L_{J_dB} &\equiv 15 \\
G_{RT_dBi} &\equiv 38 \\
N &\equiv f_c = 3 \times 10^9 \cdot Hz \\
G_{sp} &\equiv 1 \\
L_{R_dB} &\equiv 13 \\
SNR_{dt_dB} &\equiv 12 \\
c &\equiv 3 \times 10^8 \cdot \frac{m}{sec} \\
G_{RJ} &\equiv 10 \left( \frac{G_{JR_dBi}}{10} \right) = 19.9526 \\
G_J &\equiv 10 \left( \frac{L_{J_dB}}{10} \right) = 31.6228 \\
G_{RT} &\equiv 10 \left( \frac{G_{RT_dBi}}{10} \right) = 6.3096 \times 10^3 \\
SNR_{dt} &\equiv 10 \left( \frac{SNR_{dt_dB}}{10} \right) = 15.8489 \\
GRJ &\equiv 10 \left( \frac{GRJ_dBi}{10} \right) = 63.0957 \\

We compute the false target jamming-to-target signal ratio after integration of multiple pulses, \(J2S_n\) (no units) and \(J2S_{n,db}\) (dB), using Equation 11-26.
\]
\[ J_2S_n := \frac{J}{S} = 317.3265 \]

\[ J_2S_n_{\text{dB}} := 10 \log(J_2S_n) = 25.0151 \]

As discussed in Section 11.4.2, the fundamental concept of J/S assumes that a target signal is in the same resolution cell as the false target signal, which may not be the case for a support jammer! Thus, it can be challenging to accurately use J/S as a support jammer metric.

We compute the false target radar detection range, \( R_{dt} \) (meters), using Equation 11-23. We need the wavelength, \( \lambda \) (meters), for this equation and compute it using Equation 2-2.

\[ \lambda := \frac{c}{f_c} = 0.1 \text{ m} \]

\[ R_{dt} := \sqrt{\frac{P_J \cdot G_{JR} \cdot G_{RJ} \cdot \lambda^2 \cdot G_{sp} \cdot G_I}{(4 \pi)^2 \cdot \text{SNR}_{dt} \cdot N \cdot L_J}} = 3.6018 \times 10^6 \text{ m} \]

\[ R_{dt} = 3.6018 \times 10^3 \text{ km} \]

Compare to the target radar detection range, \( R_{dt_{\text{tgt}}} \) (meters), from Exercise 3-15. Wow! A little jammer goes a long way in providing an extremely long detection range.
We can obtain an understanding for why the false target jammer detection range is significantly greater than the target radar detection range by comparing the jammer effective radiated power (ERP), from Exercise 11-1, to the reflected power from the target, from Exercise 2-5. The jammer ERP is about 1.3 kW, while the reflected power from the target is about 0.06 W. The significantly higher jammer ERP contributes to a significantly longer detection range.
Exercise 11-6  In a tactical operation, a support noise jammer is to be used to screen aircraft $\sigma = 1 \text{ m}^2$ radar cross section attacking a radar system. The attacking aircraft can launch their missiles at $R_{RT} = 10$ km range. The support jammer aircraft must standoff at range $R_{RJ} = 100$ km. The radar system to be attacked is known to have the following characteristics: antenna mainbeam gain in the direction of the target $G_{RT} = 30 \text{ dBi}$ gain, peak transmit power $P_R = 1 \text{ MW}$, integrates $10 \text{ } \mu \text{sec}$ coherent pulses, signal processing gain $G_{sp} = 1$, and a matched filter receiver. To be conservative, a 0 dB S/J is required for the jammer to be effective. What effective radiated power, $ERP_J$ (Watts), does the support jammer need if the jammer noise bandwidth $B_J = 20 \text{ MHz}$? Assume the radar-related and jammer-related losses are the same, and the radar antenna gain in the direction of the support jammer is one-half the mainbeam gain.

We start by defining the values for all the terms.

$$\sigma \equiv 1 \cdot \text{m}^2 \quad G_{sp} \equiv 1 \quad R_{RT} \equiv 10 \times 10^3 \cdot \text{m} \quad R_{RJ} \equiv 100 \times 10^3 \cdot \text{m} \quad G_{RT\text{dBi}} \equiv 30 \quad P_R \equiv 1 \times 10^6 \cdot \text{W}$$

$$n_p \equiv 10 \quad \tau \equiv 1 \times 10^{-6} \cdot \text{sec} \quad SJR\_\text{dB} \equiv 0 \quad B_J \equiv 20 \times 10^6 \cdot \text{Hz}$$

We start by converting for decibels to absolute as needed. The radar antenna sidelobe gain in the direction of the jammer, $G_{RJ}$ (no units) is one-half the mainbeam gain. We compute it here as well.

$$G_{RT} := 10 \left( \frac{G_{RT\text{dBi}}}{10} \right) = 1 \times 10^3$$

$$SJR := 10 \left( \frac{0}{10} \right) = 1$$

$$G_{RJ} := \frac{G_{RT}}{2} = 500$$

The signal-to-jamming ratio (S/J), or $SJR_n$, (no units), for a standoff noise jammer is computed by inverting the noise jamming-to-target signal ratio (Equation 11-19). The jammer effective radiated power is given by Equation 11-1. We can solve this equation in terms of the jammer effective radiated power ($ERP_J$) as shown below:
Because we assume the radar-related and jammer-related losses are the same, all the loss terms can be removed from this ERP equation, as shown below:

\[ \text{ERP}_J = \frac{P_J \cdot G_J R}{L_{Jt}} = \frac{P_J \cdot G_{RT}^2 \cdot \sigma \cdot G_{sp} \cdot G_I}{4 \pi \cdot R_{RT}^4 \cdot R} \cdot \left( \frac{B_J}{B_R} \right) \cdot \left( \frac{R_{RJ}^2 \cdot L_J}{SJR_n \cdot G_{RJ}} \right) \]

We were given almost all the values for the terms in this ERP equation. We are missing the integration gain, \( G_I \) (no units) and radar bandwidth, \( B_R \) (Hertz). The integration gain for coherent pulses is given by Equation 3-35. The radar uses a matched filter bandwidth, thus the radar receiver bandwidth is given by Equation 2-15.

\[ G_I := n_p = 10 \quad B_R := \frac{1}{\tau} = 1 \times 10^6 \cdot \text{Hz} \]

\[ \frac{1}{(1 \cdot 10^{-6} \cdot \text{sec})} = 1 \times 10^6 \cdot \text{Hz} \]

We can now compute the needed jammer effective radiated power, \( \text{ERP}_J \) (Watts).

\[ \text{ERP}_J := \frac{P_R \cdot G_{RT}^2 \cdot \sigma \cdot G_{sp} \cdot G_I}{4 \pi \cdot R_{RT}^4} \cdot \left( \frac{B_J}{B_R} \right) \cdot \left( \frac{R_{RJ}^2}{SJR_n \cdot G_{RJ}} \right) = 3.1831 \times 10^4 \text{W} \]
\[
\frac{(1 \times 10^6 \text{ W}) \left[ \frac{(30)}{10} \right]^2 \cdot (1 \text{ m}^2) \cdot (1) \cdot (10)}{4\pi \left(10 \times 10^3 \text{ m} \right)^4} \cdot \left( \frac{20 \times 10^6 \text{ Hz}}{1 \times 10^6 \text{ Hz}} \right) \cdot \frac{(100 \times 10^3 \text{ m})^2}{(1) \cdot (500)} = 3.183 \times 10^4 \text{ W}
\]
Exercise 12-1  A radar system has a frequency modulated (FM) chirp waveform with a bandwidth of $B_{pc} = 100$ MHz over a transmitted pulse width $\tau = 200$ microseconds. What is the consequence of the pulse compression waveform on (a) a narrowband jammer approximately matched to the transmitted pulse width and (b) a wideband jammer approximately matched to the pulse compression modulation bandwidth?

We start by defining the values for all the terms and necessary constants.

$B_{pc} \equiv 100 \times 10^6 \cdot \text{Hz} \quad \tau \equiv 200 \times 10^{-6} \cdot \text{sec}$

(a) A narrowband jammer approximately matched to the bandwidth associated with the transmitted pulse width has a noise bandwidth, $B_J$ (Hertz), approximately equal to one over the transmitted pulse width (matched filter receiver). The bandwidth associated with the transmitted pulse width, $B_R$ (Hertz), is given in Equation 2-15.

$$B_R := \frac{1}{\tau} = 5 \times 10^3 \cdot \text{Hz}$$

As discussed in Section 5.2.2, when using pulse compression the radar receiver bandwidth is matched to the pulse compression modulation bandwidth, $B_{pc}$ (Hertz), as given in Equation 5-18. The narrowband jammer must increase its noise bandwidth to cover the wider pulse compression modulation bandwidth. The amount of increase is the ratio of the pulse compression modulation bandwidth to the bandwidth of the narrowband jammer (matched to the transmitted pulse width).

$$\frac{B_{pc}}{B_J} = 2 \times 10^4$$

Increasing the jammer noise bandwidth reduces the jammer power spectral density (power divided by bandwidth, W/Hz), and thus noise jamming power in the radar receiver (see Sections 10.3 and 11.3).

(b) A wideband jammer approximately matched to the pulse compression modulation bandwidth has a noise bandwidth, $B_J$ (Hertz), approximately equal to the pulse compression modulation bandwidth, $B_{pc}$ (Hertz).
The wideband jammer's noise bandwidth covers the pulse compression modulation bandwidth. Thus, there is no consequence of the pulse compression waveform to the wideband jammer.

\[ B_J := B_{pc} = 1 \times 10^8 \text{ Hz} \]
**Exercise 12-2** A radar system uses a variable power transmitter to provide a low probability of intercept electronic protection (EP) technique; i.e., reduce the detection range of electronic warfare (EW) receivers. The desire is to reduce the EW receiver detection range by half by reducing the transmit power. What is the necessary radar transmitter power level relative to full power?

The EW receiver detection range, $R_{dtRWR}$ (meters), is given by Equations 9-16 through 9-24. We will use Equation 9-19 since it has the fewest terms. We can use this equation to determine the detection range for the two different radar transmit powers, $P_{R1}$ (Watts) and $P_{R2}$ (Watts).

$$R_{dtRWR1} = \sqrt{\frac{P_{R1} \cdot G_{RT} \cdot G_{RWR} \cdot \lambda^2}{(4\pi)^2 \cdot S_{mindt} \cdot L_{RWR}}}$$

$$R_{dtRWR2} = \sqrt{\frac{P_{R2} \cdot G_{RT} \cdot G_{RWR} \cdot \lambda^2}{(4\pi)^2 \cdot S_{mindt} \cdot L_{RWR}}}$$

As we can see in these two equations the vast majority of terms are the same. We can represent these same terms with the term "C". We then solve each equation for "C" and set them equal to each other. We can them solve for the relationship between radar transmitter power and EW receiver detection range. All these steps are shown below.

$$C = \frac{R_{dtRWR1}^2}{P_{R1}} = \frac{R_{dtRWR2}^2}{P_{R2}}$$

$$C = \frac{P_{R2}}{P_{R1}} \cdot \frac{R_{RT2}^2}{R_{RT1}^2}$$

We can use this equation to determine the necessary radar transmitter power level relative to full power to reduce the EW receiver detection range by half.

$$\frac{P_{R2}}{P_{R1}} = \left(0.5 \times \frac{R_{dtRWR1}}{R_{dtRWR1}}\right)^2$$

$$(0.5)^2 = 0.25$$

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Exercise 12-3  A radar system has a sidelobe canceller (SLC) with an auxiliary antenna gain in the direction of the jammer $G_{AJ} = 25 \text{ dBi}$. Without the SLC the noise jamming-to-receiver thermal noise ratio $J_N/N = 20 \text{ dB}$ when the radar antenna gain in the direction of the jammer $G_{RJ} = 25 \text{ dBi}$. (a) What is the residual noise-to-receiver thermal noise ratio, $P_Z/N$ (dB)? (b) The radar-jammer geometry changes and the radar antenna gain in the direction of the jammer is 10 dB less. What is the residual noise-to-receiver thermal noise ratio, $P_Z/N$ (dB)? Compare your results with Figure 12-3.

We start by defining the values for all the terms and necessary constants and convert from decibels to absolute as needed. Note: for part (b), if the radar antenna gain in the direction of the jammer is 10 dB less, the noise jamming-to-receiver thermal noise is also 10 dB less (see Section 11.3).

$$G_{AJ \_dBi} \equiv 25 \quad G_{RJ1 \_dBi} \equiv 25 \quad JNR1 \_dB \equiv 20$$

$$G_{RJ2 \_dBi} := G_{RJ1 \_dBi} - 10 = 15$$

$$JNR2 \_dB := JNR1 \_dB - 10 = 10$$

First convert from decibels to absolute as needed.

$$G_{AJ} := 10 \left( \frac{G_{AJ \_dBi}}{10} \right) = 316.2278$$

$$G_{RJ1} := 10 \left( \frac{G_{RJ1 \_dBi}}{10} \right) = 316.2278$$

$$JNR1 := 10 \left( \frac{JNR1 \_dB}{10} \right) = 100$$

$$G_{RJ2} := 10 \left( \frac{G_{RJ2 \_dBi}}{10} \right) = 31.6228$$

$$JNR2 := 10 \left( \frac{JNR2 \_dB}{10} \right) = 10$$
(a) We compute the residual noise-to-receiver thermal noise ratio, $P_{ZNR}$ (no units and dB), using Equation 12-2.

$$P_{ZNR1} := \frac{J_{NR1} \left( \frac{G_{AJ}}{G_{RJ1}} + 1 \right) + 1}{J_{NR1} \left( \frac{G_{AJ}}{G_{RJ1}} \right) + 1} = 1.9901$$

$$10 \log (P_{ZNR1}) = 2.9887$$

This value is the same as what we can read from Figure 12-3 for the noise jamming-to-receiver thermal noise ratio and the ratio of the auxiliary antenna gain in the direction of the jammer and the radar antenna gain gain in the direction of the jammer for part (a).

(b) We compute the residual noise-to-receiver thermal noise ratio, $P_{ZNR}$ (no units and dB), using Equation 12-2.
\[
PZNR2 := \frac{\frac{G_{AJ}}{G_{RJ2}} + 1}{JNR2 \cdot \left(\frac{G_{AJ}}{G_{RJ2}} + 1\right)} = 1.099
\]

10 \log (PZNR2) = 0.41

This value is the same as what we can read from Figure 12-3 for the noise jamming-to-receiver thermal noise ratio and the ratio of the auxiliary antenna gain in the direction of the jammer and the radar antenna gain gain in the direction of the jammer for part (b).

\[
JNR2_{\text{dB}} = 10 \quad 10 \log \left(\frac{G_{AJ}}{G_{RJ2}}\right) = 10
\]

\[
G_{AJ}_{\text{dBi}} - G_{RJ2}_{\text{dBi}} = 10
\]

\[
= 25 - 15 = 10
\]
Exercise 12-4  What are the cancellation ratios (CR) for the two radar-jammer conditions in Exercise 12-3 above? Compare your results with Figure 12-4.

We start by defining the values for all the terms and necessary constants.

\[ \begin{align*} 
G_{AJ\_dBi} & = 25 \\
G_{RJ1\_dBi} & = 25 \\
JNR1\_dB & = 20 \\
G_{RJ2\_dBi} & := G_{RJ1\_dBi} - 10 = 15 \\
JNR2\_dB & := JNR1\_dB - 10 = 10 \\
\end{align*} \]

First convert from decibels to absolute as needed.

\[ \begin{align*} 
G_{AJ} & := 10 \left( \frac{G_{AJ\_dBi}}{10} \right) = 316.2278 \\
G_{RJ1} & := 10 \left( \frac{G_{RJ1\_dBi}}{10} \right) = 316.2278 \\
JNR1 & := 10 \left( \frac{JNR1\_dB}{10} \right) = 100 \\
G_{RJ2} & := 10 \left( \frac{G_{RJ2\_dBi}}{10} \right) = 31.6228 \\
JNR2 & := 10 \left( \frac{JNR2\_dB}{10} \right) = 10 \\
\end{align*} \]

(a) We compute the cancellation ratio, CR (no units and dB), using Equation 12-5.
\[ \frac{(JNR1 + 1) \cdot \left( \frac{G_{AJ}}{G_{RJ1}} + 1 \right)}{JNR1 \cdot \left( \frac{G_{AJ}}{G_{RJ1}} + 1 \right) + 1} = 50.7512 \]

This value is the same as what we can read from Figure 12-4 for the noise jamming-to-receiver thermal noise ratio and the ratio of the auxiliary antenna gain in the direction of the jammer and the radar antenna gain gain in the direction of the jammer for part (a).

\[ 10 \log (CR1) = 17.0545 \]

\[ = 10 \cdot \log (50.75124378109453) = 17.0545 \]

\( JNR1_{dB} = 20 \)

\[ 10 \log \left( \frac{G_{AJ}}{G_{RJ1}} \right) = 0 \]

\[ = 10 \cdot \log \left( \frac{25}{10} \right) = 0 \]

\[ G_{AJ\_dB} - G_{RJ1\_dB} = 0 \]

\[ = 25 - 25 = 0 \]

(b) We compute the cancellation ratio, CR (no units and dB), using Equation 12-5.
\[
\begin{align*}
\text{CR}2 &:= \frac{(JNR2 + 1) \cdot \left( \frac{G_{AJ}}{G_{RJ2}} + 1 \right)}{JNR2 \cdot \left( \frac{G_{AJ}}{G_{RJ2}} + 1 \right) + 1} = 10.009 \\
(10 + 1) \cdot 10 \cdot \left( \frac{25}{10} \right) + 1 &\quad = 10.009 \\
10 \cdot \left( \frac{25}{10} + 1 \right) + 1 &\quad = 10.009 \\
10 \log \left( \text{CR}2 \right) &\quad = 10.0039 \\
&\quad = 10 \cdot \log (10.009009009009009009) = 10.0039
\end{align*}
\]

This value is the same as what we can read from Figure 12-4 for the noise jamming-to-receiver thermal noise ratio and the ratio of the auxiliary antenna gain in the direction of the jammer and the radar antenna gain gain in the direction of the jammer for part (a).

\[
\begin{align*}
\text{JNR2\_dB} = 10 &\quad 10 \log \left( \frac{G_{AJ}}{G_{RJ2}} \right) = 10 \\
&\quad = 10 \cdot \log \left( \frac{25}{10} \right) = 10 \\
G_{AJ\_dBi} - G_{RJ2\_dBi} = 10 &\quad = 25 - 15 = 10
\end{align*}
\]
Exercise 12-5  The radar system from Exercises 12-3 and 12-4 above has a received target signal power $S = -124$ dBW, integration gain $G_I = 20$ dB, signal processing gain $G_{sp} = 1$, and receiver thermal noise power $N = -120$ dBW. (a) What is the target signal-to-noise ratio after integration, $(S/N)_n$ (dB)? For the initial radar-jammer geometry from Exercise 12-3: (b) what is the target signal-to-interference ratio after integration, $(S/I)_n$ (dB)? (c) What is the target signal-to-interference ratio after integration with the SLC, $(S/I)_n$ (dB)? For the radar-jammer geometry from Exercise 12-3 part (b): (d) what is the target signal-to-interference ratio after integration, $(S/I)_n$ (dB)? (e) What is the target signal-to-interference ratio after integration with the SLC, $(S/I)_n$ (dB)?

We start by defining the values for all the terms and necessary constants.

\[
\begin{align*}
S_{dBW} &= -124 \\
G_{I dB} &= 20 \\
G_{sp} &= 1 \\
N_{dBW} &= -120 \\
CR_{1 dB} &= 17.0545 \\
CR_{2 dB} &= 10.0039 \\
JNR_{1 dB} &= 20 \\
JNR_{2 dB} &= 10 \\
\end{align*}
\]

Convert from decibels to absolute as needed.

\[
\begin{align*}
S &= 10^{\left(\frac{S_{dBW}}{10}\right)} \cdot 1 W = 3.9811 \times 10^{-13} W \\
N &= 10^{\left(\frac{N_{dBW}}{10}\right)} \cdot 1 W = 1 \times 10^{-12} W \\
G_I &= 10^{\left(\frac{G_{I dB}}{10}\right)} = 100 \\
CR_{1} &= 10^{\left(\frac{CR_{1 dB}}{10}\right)} = 50.7516 \\
CR_{2} &= 10^{\left(\frac{CR_{2 dB}}{10}\right)} = 10.009 \\
\end{align*}
\]

As we will see below, the equations used to solve this exercise need the noise jamming power, $J_N$ (Watts). We will compute the noise jamming power based on the provided noise jamming-to-receiver thermal noise ratio, JNR (no units and JNR_{dB} (dB), and radar receiver thermal noise power, N (no units) and N_{dBW} (dBW). We will start with the algebraic form of the JNR equation and finish with the logarithmic form of the $J_N$ equation. We will convert from decibels to absolute as needed to obtain the values we need.
JNR = \frac{J_N}{N} \quad J_N = N \cdot JNR

J_{N1\_dBW} := N\_dBW + JNR1\_dB = -100

J_{N2\_dBW} := N\_dBW + JNR2\_dB = -110

J_{N1} := 10 \left( \frac{-100}{10} \right) \cdot W = 1 \times 10^{-10} W

\frac{(-100)}{10} \cdot 1 \cdot W = 1 \times 10^{-10} W

J_{N2} := 10 \left( \frac{-110}{10} \right) \cdot W = 1 \times 10^{-11} W

(a) We compute the target signal-to-noise ratio after integration of multiple pulses, SNR_n (no units and dB) using Equation 3-38.

SNR_n := \frac{S \cdot G_{sp} \cdot G_I}{N} = 39.8107

10 \log (SNR_n) = 16

(b) We compute the target signal-to-interference ratio after integration of multiple pulses, SIR_n (no units and dB), using Equation 11-17.

SNR_n := \frac{S \cdot G_{sp} \cdot G_I}{N} = 39.8107 \cdot W

= 10 \left[ \frac{-124}{10} \right] \cdot 1 \cdot 100 = 39.8107 W

10 \log (SNR_n) = 16

= 10 \cdot \log (39.8107170553497) = 16
The noise jammer results in an extremely small \( (S/I)_n \).

(c) We compute the post-cancellation target signal-to-interference ratio after integration of multiple pulses, \( SIR_{n\_CR} \) (no units and dB), using Equation 12-7.

\[
SIR_{n\_CR} := \frac{S \cdot G_{sp} \cdot G_I}{J_{N1} + N} = 20.0045
\]

\[
10 \log (SIR_{n\_CR}) = 13.0113
\]

The SLC has greatly reduced the noise jamming resulting in a \( (S/I)_n \) almost as high as the \( (S/N)_n \).

(d) We compute the target signal-to-interference ratio after integration of multiple pulses, \( SIR_n \) (no units and dB), using Equation 11-17.
\[
\text{SIR}_{n2} := \frac{S \cdot G_{sp} \cdot G_I}{J_{N2} + N} = 3.6192
\]

\[
10 \log (\text{SIR}_{n2}) = 5.5861
\]

The noise jammer results in a small \((S/I)_n\).

(e) We compute the post-cancellation target signal-to-interference ratio after integration of multiple pulses, \(\text{SIR}_{n_{\text{CR}}}\) (no units and dB), using Equation 12-7.

\[
\text{SIR}_{n_{\text{CR}}} := \frac{S \cdot G_{sp} \cdot G_I}{(J_{N2} + N)_{\text{CR2}}} = 36.2241
\]

\[
10 \log (\text{SIR}_{n_{\text{CR}}}) = 15.59
\]

The SLC has almost eliminated the noise jamming resulting in a \((S/I)_n\) almost equal to the \((S/N)_n\).
**Exercise 13-1** What is the radar line of sight for a radar height \( h_R = 10 \) meters and a target height \( h_T = 2000 \) meters? Use a refraction factor \( k_r = 4/3 \).

We start by defining the values for all the terms and necessary constants.

\[
\begin{align*}
h_R &\equiv 10 \cdot m \\
h_T &\equiv 2000 \cdot m \\
k_r &\equiv \frac{4}{3} \\
R_E &\equiv 6371 \times 10^3 \cdot m
\end{align*}
\]

The radar line of sight (LOS), \( R_{\text{LOS}} \) (meters), is given by Equation 13-7 based on the clutter horizon, \( R_C \) (meters), as given in Equation 13-5, and the Target horizon, \( R_T \) (meters), as given in Equation 13-6.

\[
R_{hC} := \sqrt{2 k_r \cdot R_E \cdot h_R + h_R^2} = 1.3034 \times 10^4 \ m
\]

\[
= \sqrt{2 \cdot \left(\frac{4}{3}\right) \cdot (6371 \cdot 10^3 \cdot m) \cdot (10 \cdot m) + (10 \cdot m)^2} = 1.3034 \times 10^4 \ m
\]

\[
R_{hT} := \sqrt{2 k_r \cdot R_E \cdot h_T + h_T^2} = 1.8434 \times 10^5 \ m
\]

\[
= \sqrt{2 \cdot \left(\frac{4}{3}\right) \cdot (6371 \cdot 10^3 \cdot m) \cdot (2000 \cdot m) + (2000 \cdot m)^2} = 1.8434 \times 10^5 \ m
\]

\[
R_{\text{LOS}} := R_{hC} + R_{hT} = 1.9738 \times 10^5 \ m
\]

\[
1.3034 \times 10^4 \ m + 1.8434 \times 10^5 \ m = 1.9737 \times 10^5 \ m
\]

\[
R_{\text{LOS}} = 197.3782 \cdot km
\]
**Exercise 13-2** Fifteen ground-based radars stretched across the northern border of the United States will give coverage at 1400 meter altitude and above. How many ground-based radars will be required to extend coverage down to 60 meters? Use a refraction factor $k_r = 4/3$.

We start by defining target altitudes, number of radars, the radius of the earth, and refraction factor.

\[
\begin{align*}
h_1 & = 1400 \text{ m} \\
h_2 & = 60 \text{ m} \\
N_{\text{radar}_1} & = 15 \\
R_E & = 6371 \times 10^3 \text{ m} \\
k_r & = \frac{4}{3}
\end{align*}
\]

We solve this problem in multiple steps. First, we compute the radar horizon, $R_{h1}$ (m), associated with the coverage of targets at 1400 meter altitude and above, using Equation 13-2. Note: the radar is assumed to have an antenna height of zero (0). Next, we find the length of the northern border of the US, Border (meters), by multiplying two times the radar horizon by the number of radars needed to given coverage at 1400 meter, $N_{\text{radar}_1}$ (no units). The two comes from the radar horizon is the radius of a circle, and one radar covers twice the radius (diameter) of the circle. A figure showing the radar horizons along the border and all the equations and values are shown below:

\[
R_{h1} := \sqrt{2 \cdot k_r \cdot R_E \cdot h_1 + h_1^2} = 1.5423 \times 10^5 \text{ m}
\]

\[
= \sqrt{2 \cdot \left(\frac{4}{3}\right) \cdot (6371 \times 10^3 \text{ m}) \cdot (1400 \text{ m}) + (1400 \text{ m})^2} = 1.5423 \times 10^5 \text{ m}
\]

\[
\text{Border} := N_{\text{radar}_1} \cdot (2 \times R_{h1}) = 4.6269 \times 10^6 \text{ m}
\]
Then we find the radar horizon associated with the coverage of targets down to 60 meters, $R_{h2}$ (meters). Finally, we find the number of radars required to extend coverage down to 60 meters, $N_{\text{radar2}}$ (no units) by dividing the length of the northern border by the radar horizon associated with coverage down to 60 meters. All the equations and values are shown below:

$$R_{h2} := \sqrt{2 \cdot k_r \cdot R_E \cdot h_2 + h_2^2} = 3.1927 \times 10^4 \text{ m}$$

$$= \sqrt{2 \cdot \left(\frac{4}{3}\right) \cdot (6371 \cdot 10^3 \text{ m}) \cdot (60 \text{ m}) + (60 \text{ m})^2} = 3.1927 \times 10^4 \text{ m}$$

$$N_{\text{radar2}} := \frac{\text{Border}}{2 \times R_{h2}} = 72.4597$$

$$= \frac{4.6269 \times 10^6 \text{ m}}{2 \cdot (3.1927 \times 10^4 \text{ m})} = 72.4606$$

Because radar systems only come in integer numbers, we round this result up to 73.
**Exercise 13-3** How many airborne pulse Doppler radars flying at 6000 meters are required to cover the United States northern border against targets flying down to 30 meters? Use a refraction factor $k_r = 4/3$.

We start by defining the values for all the terms and necessary constants. Note: The length of the US northern border, $Border$ (meters), was computed in Exercise 13-2.

$$h_R \equiv 6000 \text{ m} \quad h_T \equiv 30 \text{ m} \quad k_r = \frac{4}{3} \quad R_E \equiv 6371 \times 10^3 \text{ m} \quad Border \equiv 4.6269 \times 10^6 \text{ m}$$

We will solve for the number of radars, $N_{radar}$ (no units), by first computing the radar line of sight (LOS), $R_{LOS}$ (meters), using Equation 13-7. The radar LOS is a function of the clutter horizon, $R_{hC}$ (meters) Equation 13-5, and target horizon, $R_{hT}$ (meters) Equation 13-6. Next, we find the number of radars by dividing the length of the northern border of the US, $Border$ (meters), by two times the radar LOS. The two comes from the radar LOS is the radius of a circle, and one radar covers twice the radius (diameter) of the circle. All the equations and values are shown below:

$$R_{hC} := \sqrt{2k_r \cdot R_E \cdot h_R + h_R^2} = 3.1933 \times 10^5 \text{ m}$$

$$= \sqrt{2 \cdot \left( \frac{4}{3} \right) \cdot (6371 \times 10^3 \text{ m}) \cdot (6000 \text{ m}) + (6000 \text{ m})^2} = 3.1933 \times 10^5 \text{ m}$$

$$R_{hT} := \sqrt{2k_r \cdot R_E \cdot h_T + h_T^2} = 2.2576 \times 10^4 \text{ m}$$

$$= \sqrt{2 \cdot \left( \frac{4}{3} \right) \cdot (6371 \times 10^3 \text{ m}) \cdot (30 \text{ m}) + (30 \text{ m})^2} = 2.2576 \times 10^4 \text{ m}$$

$$R_{LOS} := R_{hC} + R_{hT} = 3.4191 \times 10^5 \text{ m}$$

$$3.1933 \times 10^5 \text{ m} + 2.2576 \times 10^4 \text{ m} = 3.4191 \times 10^5 \text{ m}$$

$$N_{radar} := \frac{Border}{2 \times R_{LOS}} = 6.7663$$

$$= \frac{4.6269 \times 10^6 \text{ m}}{2 \cdot (3.4191 \times 10^5 \text{ m})} = 6.7663$$

Because radar systems only come in integer numbers, we round this result up to 7.
**Exercise 13-4** A radar system has the following characteristics: carrier frequency $f_c = 9.5$ GHz and radar system losses without atmospheric attenuation $L_R = 6$ dB. A very low altitude target comes above the radar horizon at a range $R_{RT} = 30$ km. (a) What is the atmospheric attenuation loss, $L_a$ (dB)? (b) What is the radar system losses including atmospheric attenuation, $L_R$ (dB)?

We start by defining the values for all the terms.

$$L_R_{\text{dB}} \equiv 6 \quad R_{RT_{\text{km}}} \equiv 30$$

(a) Tropospheric atmospheric attenuation is shown in Figure 13-8. At $9.5$ GHz and $0$ km target altitude (very low altitude target just over the horizon), the topospheric atmospheric attenuation is approximately $0.01$ dB/km. The atmospheric attenuation loss, $L_a_{\text{dB}}$ (dB), is the product of the atmospheric attenuation (dB/km) and the total propagation path length. The total propagation path length is two times the radar-to-target range, one range from the radar to the target and one range back to the radar. This calculation is shown below:

$$L_a_{\text{dB}} := 0.01 \cdot (2 \times R_{RT_{\text{km}}}) = 0.6$$

Note: this is one the extremely rare times we use decibels and absolute values in the same equation. We can do this because of the units of the atmospheric attenuation (dB/km).

(b) The radar system losses including atmospheric attenuation is the sum (since all terms are in decibels) of the radar system losses without atmospheric attenuation and atmospheric attenuation loss.

$$L_R_{\text{dB}} + L_a_{\text{dB}} = 6.6$$

Some may think this $0.6$ dB increase is not a big deal, however we know from detection theory in Chapter 3 how much $P_d$ and/or $P_{fa}$ can change with slight changes in target signal-to-noise ratio (S/N). And of course, S/N changes linearly with $L_R$. This is shown numerically below.

$$\frac{L_a_{\text{dB}}}{10} = 1.1482$$

$$= 10 \left( \frac{0.6}{10} \right) = 1.1482$$
Exercise 13-5 Using electron densities of $10^4$ and $10^5$/cm$^3$, calculate some ionospheric effects (attenuation, refraction, and polarization rotation) for a microwave frequency $f_c = 3$ GHz.

We start by defining the values for all the terms.

The radar frequency is defined in megaHertz, as the equations need it in those units.

\[ N_{e1} \equiv 10^4 \quad N_{e2} \equiv 10^5 \quad f_c \equiv 3000 \]

Ionospheric attenuation, $\alpha$ (dB/Nmi), is given by Equation 13-9. Ionospheric index of refraction, $\eta$, is given by Equation 13-10. Ionospheric polarization rotation, $\Omega$ (degrees), is given by Equation 13-11.

\[
\begin{align*}
\alpha_1 &:= 4 \times 10^{-3} \cdot \frac{N_{e1}}{f_c^2} = 4.4444 \times 10^{-6} \\
\alpha_2 &:= 4 \times 10^{-3} \cdot \frac{N_{e2}}{f_c^2} = 4.4444 \times 10^{-5} \\
\eta_1 &:= \left(1 - \frac{N_{e1}}{10^4 \cdot f_c^2}\right)^2 = 0.9999998 \\
\eta_2 &:= \left(1 - \frac{N_{e2}}{10^4 \cdot f_c^2}\right)^2 = 0.9999978 \\
\Omega &:= \frac{6.3 \times 10^{20}}{f_c^2} = 7 \times 10^{13}
\end{align*}
\]
Exercise 13-6  What is the far field range of the Arecibo antenna from Section 7.7 on its boresight?

First we need to define the diameter and frequency of the Arecibo antenna. This information is given in the discussion of Ionospheric Radars, Section 7.7. Also define the needed constants.

\[ f_c \equiv 430 \times 10^6 \, \text{Hz} \quad D \equiv 305 \cdot \text{m} \quad c \equiv 3 \times 10^8 \cdot \frac{\text{m}}{\text{sec}} \]

The range to the far field, \( R_{ff} \) (meters), is given by Equation 13-13. We need the wavelength, \( \lambda \) (meters), and will compute it using Equation 2-2.

\[
\lambda := \frac{c}{f_c} = 0.6977 \, \text{m} \\
\]

\[
R_{ff} := \frac{D^2}{\lambda} = 1.3334 \times 10^5 \, \text{m} \\
\]

\[ R_{ff} = 133.3358 \, \text{km} \]
Exercise 13-7 The space track radar in Florida has an antenna aperture dimension of 55 meters at UHF ($\lambda = 0.6$ meters). What is the range to its far field?

We start by defining the values for all the terms.

$$D \equiv 55 \cdot m \quad \quad \quad \lambda \equiv 0.6 \cdot m$$

The range to the far field, $R_{ff}$ (meters), is given by Equation 13-13.

$$R_{ff} := \frac{D^2}{\lambda} = \frac{55 \cdot m}{0.6 \cdot m} = 5.0417 \times 10^3 \, m$$

$$R_{ff} = 5.0417 \cdot km$$
**Exercise 13-8** A homemade telescope has an aperture of 0.2 meters. What is the range to its far field? (Take optical wavelengths to be 0.5 micrometers)

We start by defining the values for all the terms.

\[ D = 0.2 \cdot \text{m} \quad \lambda = 0.5 \times 10^{-6} \cdot \text{m} \]

The range to the far field, \( R_{ff} \), is given by Equation 13-13.

\[
R_{ff} := \frac{D^2}{\lambda} = \frac{(0.2 \cdot \text{m})^2}{(0.5 \times 10^{-6} \cdot \text{m})} = 8 \times 10^4 \text{ m}
\]

\( R_{ff} = 80 \cdot \text{km} \)